Targeted, One-to-one Instruction in Whole-number Arithmetic: A Framework of Key Elements

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Presented in fulfilment of the requirements for the degree of Doctor of Philosophy

January, 2016

I certify that the work presented in this thesis is, to the best of my knowledge and belief, original, except as acknowledged in the text, and that the material has not been submitted, either in whole or in part, for a degree at this or any other university.

I acknowledge that I have read and understood the University's rules, requirements, procedures and policy relating to my higher degree research award and to my thesis. I certify that I have complied with the rules, requirements, procedures and policy of the University (as they may be from time to time).

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Abstract

In Australia, although there has been strong advocacy for individualized intervention programs, there is a limited research literature available that focuses on teacher-student interactions and teaching practices related to one-to-one instruction. This investigation seeks to address that gap. Its aim is to identify and illuminate the nature of Key Elements of one-to-one instruction that expert tutors use when interacting in intensive, one-to-one instruction of whole-number arithmetic with Years 3 and 4 students. A Key Element is a micro-instructional strategy that is the smallest unit of analysis of highly interactive one-to-one instruction.

The investigation draws on data collected within the framework of the Mathematics Intervention Specialist Program (Wright, Ellemor-Collins & Lewis, 2011). From this source, approximately 33 hours of video recordings of teaching sessions involving four teachers and six students were analysed.

The theoretical perspective underpinning the investigation is interpretative. Within this perspective, a phenomenological approach was used to gain insight into the essence of the Key Elements of one-to-one intervention teaching. A standard method for analysing the data, that is, "close observation" (Van Manen, 1997, p. 68), in which the Key Elements are viewed as the central phenomenon requiring exploration and understanding, was employed. The analytical techniques described by Van Manen (1990, 1997), and further elaborated as procedures for phenomenological analysis by Hycner (1999), were applied. As well, the investigation utilised methodological approaches described by Cobb and Whitenack (1996), and by Powell, Francisco, and Maher (2003), for analysing large sets of video recordings.

Twenty-five Key Elements were identified and for each, a deeply layered description was developed. As well, a comprehensive framework for analysing one-to-one instruction was conceptualised. The framework shows how Key Elements can be used to analyse intensive, one-to-one instruction in whole-number arithmetic.

The investigation advances understanding about teacher-student interactions and teaching practice in intensive, one-to-one interventions. Understanding the Key Elements leads to more effective ways to characterise the instructional strategies that teachers utilise in one-to-one intervention teaching. The framework developed constitutes an extension of the current body of theoretical knowledge about targeted one-to-one intensive intervention in whole-number arithmetic. It will inform teachers who are working with low-attaining students by providing

useful information about teacher-student interaction in mathematical interventions (Tran & Wright, 2014b).

Acknowledgements

The thesis could not have been completed without the great support from so many people over the last four years. I am hugely indebted to all those concerned.

Most of all, I wish to offer my most heartfelt thanks to my supervisors, Adjunct Professor Bob Wright, Professor Martin Hayden and Dr Geoff Woolcott, for their invaluable mentoring, enthusiastic encouragement and useful critiques of my research work. I owe my deepest gratitude to Adjunct Professor Bob Wright who has encouraged, guided and supported me throughout my PhD studies. He encouraged me to choose the topic for my thesis and has taken me into an area of mathematics education that I have found fascinating and about which I have become passionate. A very special thanks goes to Professor Martin Hayden for his generosity with his time and sage advice. His insightful comments on the structure of my thesis and on my approach to its documentation helped enormously throughout my candidature. My intellectual debt to Dr Geoff Woolcott is equally significant. He carefully read my drafts and made meticulous comments that greatly enriched my work. Each has been extraordinarily tolerant and supportive, and I am deeply indebted to each of them.

Words are powerless to express my gratitude to Associate Professor Sharon Perry who has provided enormous support in my choosing the methodology and in approaching the data analysis for my investigation. She has been inspirational, and her kind and generous support, both academically and personally, will long be remembered.

I am deeply grateful to Professor Anne Graham who reviewed my methodology chapter and gave me insightful comments and suggestions.

I also express my appreciation to the following people who have supported me in one way or another throughout candidature: to David Ellemor-Collins, who acted as an independent auditor for my data collection and analysis; to Dr Drageset Ove Gunnar, who discussed my research and shared many ideas relevant to thesis topic; to Paul Perversi, who has given me constructive comments and warm encouragement; to Di Davies, for reading my thesis and providing much-needed help with my English; to fellow "Maths Edders" Andrea, Vicki, Lucy and Rumi, for their good humour and warmth of encouragement; and to my fellow PhD candidates from Vietnam, including Kim Anh, Hieu, Chinh, and Gam, for their sympathy in developing a doctoral thesis in English and for their much-appreciated social support.

I express my gratitude to the Government of Vietnam for giving me a scholarship to study in Australia. I also thank Southern Cross University for the additional financial support provided, and I express special thanks to my colleagues in the School of Education at Southern Cross University.

Last but not least, I express my very deep appreciation of the love and support provided to me by my family. My family's belief in me, and their resilience in dealing with the many sacrifices of candidature, enabled me to survive the journey.

Conference Attendances and Publications

During my PhD journey, I have attended national and international conferences, at some of which I presented papers. As well, I contributed a chapter to the Teacher Noticing Monograph, which is in progress. Details are as follow.

Conference		
Conference names	**Presentation title**	**Further details**
International Group for the Psychology of Mathematics Education (PME) 2014	Using an experimental framework of key elements to parse one-to-one, targeted intervention teaching in whole-number arithmetic	Tran, L.T., & Wright, R.J. (2014). In C. Nicol, S. Oesterle, P. Liljedahl & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 5, pp. 265-272). Vancouver, Canada: PME.
Mathematics Education Research Group of Australasia (MERGA) 2014	Beliefs of teachers who teach intensive one-to-one intervention about links to classroom teaching	Tran, L.T., & Wright, R.J. (2014). In J. Anderson, M. Canvanagh & A. Prescott (Eds.), *Proceedings of the 37th annual conference of Mathematics Education Research Group of Australasia*, (pp. 621-629). Sydney, Australia: MERGA.
Monograph		
Monograph title	**Chapter title**	**Further details**
Teacher noticing monograph	Teachers' professional noticing from a perspective of Key Elements of intensive, one-to-one intervention	Tran, L. T., & Wright, R. J. (accepted for publication). In E. O. Schack, F. Molly & J. Wilhelm (Eds.), *Teacher noticing monograph*. New York: Springer.

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Abbreviations

AAMT	Australian Association Mathematics Teachers
ACARA	Australian Curriculum, Assessment and Reporting Authority
ACER	Australian Council for Educational Research
CMIT	Count Me in Too
CMITT	Count Me in Too Indigenous
COAG	Council of Australian Governments
EMU	Extending Mathematical Understanding Intervention
GRIN	Getting ready in Numeracy
LIEN	Learning in Early Numeracy
NIP	Numeracy Intervention Project
NIRP	Numeracy Intervention Research Project
RtI	Response to Intervention
SINE	Success in Numeracy Education
TEN	Best Start Targeted Early Numeracy
TOWN	Taking off with Numeracy
VIBA	Videotaped Interview-based Assessment
ZPD	Zone Proximal Development

Chapter 1 – **Introduction**

This investigation concerns the available approaches to address the difficulties in learning mathematics experienced by children 9 to 10 years of age. In particular, the investigation draws on aspects of the Mathematics Intervention Specialist Program (Wright, Ellemor-Collins, & Lewis, 2011), a Project that aims to build teacher expertise related to intensive, one-to-one instruction in teaching number and arithmetic across the P-4 years. The investigation aims to identify and illuminate the nature of Key Elements of one-to-one instruction that expert tutors use when interacting in intensive, one-to-one instruction of whole-number arithmetic with Years 3 and 4 students. A Key Element is a micro-instructional strategy that is the smallest unit of analysis of highly interactive one-to-one instruction. A more complete definition of a Key Element will follow in Section 1.2 of this chapter.

The investigation is important because a better understanding of the nature of Key Elements might lead to more effective ways to characterise the spectrum of instructional methods teachers use. Such an understanding might also inform teachers working with low-attaining students by providing them with information about how teachers and students interact in mathematical interventions, which in turn may illuminate how particular practices influence students' learning outcomes.

This chapter includes the background, context and rationale for the investigation, as well as the research problem and a brief description of a conceptual framework that informs the investigation. The significance of the investigation is then outlined, and a brief description of the methodology is presented. Finally, an outline of the thesis is provided.

1.1 Context and Rationale for the Investigation

1.1.1 Numeracy, School Mathematics, Whole-number Arithmetic and 'Learning Difficulties'

Before discussing the context and rationale for the investigation, the following terms are explained in order to clarify how they are used in the study: numeracy, school mathematics, whole-number arithmetic and 'learning difficulties'. Numeracy is not identical to school mathematics but school mathematics underpins numeracy (AAMT, 1997, p. 11). The concepts of numeracy and school mathematics seem to overlap in meaning, but they differ in how they are applied. According to Milton (2000, p. 110), school mathematics involves a set of mathematical skills and concepts to be learned and applied. Numeracy, on the other hand, links up the conceptual understanding in mathematics and the capacity to use mathematical knowledge suitably to solve real life problems. Following Milton (2000), the term 'numeracy' is used to describe mathematics teaching and learning in school and can be interchanged with the term 'mathematics'. The term 'whole-number arithmetic', as used in the present investigation, refers to counting and strategies for carrying out basic operations (addition, subtraction, multiplication and division) with whole-numbers, with a focus on mathematics at the levels of Years 3 and 4.

There is no agreed definition of 'learning difficulties' in the subject of mathematics (Louden et al., 2000; Purdie & Ellis, 2005). Terms that are often used to refer to students with learning difficulties include 'students with special needs', 'students at risk', 'dyslexic students', 'low-attaining students' and 'students with dyscalculia'. Due to the lack of clarity about its definition, it may be very difficult to estimate the numbers of students with learning difficulties. Consequently, there is a lack of consistency in the application of effective teaching practices for students with learning difficulties (Ellis, 2005; Louden et al., 2000; Purdie & Ellis, 2005). In the present investigation, 'learning difficulties' will refer throughout to 'learning difficulties in number and early arithmetic', as indicated by a student being in the lowest 25% of attainment levels in number and early arithmetic for their grade level. These students are considered to be the most at risk of failing to learn mathematics successfully. Where terms such as 'learning disabilities in numeracy', 'learning difficulties in numeracy', 'students with special needs' and 'dyscalculia' occur in the literature referred to below, these terms have been taken to mean 'learning difficulties in number and early arithmetic', unless it is evident that a different meaning is intended to be applied to the terms.

1.1.2 Identification of Mathematical Learning Difficulties

For teachers, researchers and educators in Australia whose main concern is students with learning difficulties, particularly in mathematics, the current high rate of students experiencing difficulties in learning mathematics is a matter of considerable concern. Information drawn from the results of national surveys (e.g., Milton, 2000), and national reports (ACARA, 2013; COAG Reform Council, 2013; National Numeracy Review Report, 2008) indicates that a high proportion of students achieve below the national minimum standard in numeracy. The present investigation is situated in the context of mathematics education in Australia and, more specifically, in the development of mathematics interventions in primary education. A discussion of the context and rationale of the investigation, therefore, focuses on issues concerning students' performance in mathematics at primary school level in Australia and governmental actions in response to those issues.

An increased emphasis on numeracy development has been evident in Australia since the 1990s. School education systems are increasingly seeking for innovative approaches to advance students' learning of mathematics in order to provide them with more opportunities for success in mathematics and to improve their attitudes toward learning mathematics (Australian Education Council, 1991). *The Adelaide Declaration on national goals for schooling in the twenty-first century* (MCEETYA, 1999), endorsed by all state and territory Education Ministers, established the goals for Australian schooling for the ten-year period, 1999–2008. This statement of national goals marked an important milestone in Australian education due to its focus on numeracy and literacy as the initial priority areas of schooling. The *Adelaide Declaration* states that all students should have "attained the skills of numeracy and English literacy; such that, every student should be numerate, able to read, write, spell and communicate at an appropriate level" (MCEETYA, 1999, p. 4). Accordingly, the Commonwealth Government's policy entitled *Numeracy, a priority for all: Challenges for Australian schools* (Department of Education Training and Youth Affairs, 2000, p. 12) states that:

> …numeracy, like literacy, provides key enabling skills for individuals to participate successfully in schooling. Furthermore, numeracy equips students for life beyond school, in providing access to further study or training, to personal pursuits and to participation in the world of work and in the wider community.

Since the late 1990s, research on students' performance in mathematics in Australian schools has addressed the problem of mathematics learning difficulties among school students. For example, in a national survey of 377 schools in Australia, referred to as *Mapping the territory* (Milton, 2000, p. 121), a majority of school principal respondents (53%) informed that the percentage of their students having difficulties learning mathematics is from 10% to 30%. Further, 35% of principals reported that less than 10% of their students have such difficulties, 6% of principals reported that more than 30% of their students had such difficulties, and 1% of principals admitted that they did not know if their students have such difficulties. The survey's results point out that there is remarkably large proportion of the number of Australian students with learning numeracy difficulties. However, only 14% of principals reported that they have programs in place to assist such students in mathematics learning. This strongly indicates that there is not enough emphasis in schools on addressing mathematics difficulties and on supporting the students who have these learning difficulties.

Students' performance on numeracy over a seven-year period (2000–06), in relation to the percentage of students not meeting the national numeracy standard, is now reviewed.

Table 1.1 Percentage of Years 3, 5 and 7 students not meeting the national numeracy benchmarks in Australia, 2000-06

Year	Year 3	Year 5	Year 7
2000	7.3	10.4	
2001	6.1	10.4	18.0
2002	7.2	10.0	16.5
2003	5.8	9.2	18.7
2004	6.3	8.8	17.9
2005	5.9	9.2	18.2
2006	7.0	9.7	20.3

Source: Extracted from MCEETYA, 2006, pp. 14, 25, 36.

Note: Numeracy results not reported for 1999 and Year 7 in 2000.

Table 1.1 indicates that numeracy achievement at the national level in Year 3 fluctuated slightly, remaining almost the same over the period of 2000–06. This trend occurred similarly for Year 5 and Year 7. However, the results also show that the proportion of students not meeting the national numeracy benchmarks becomes significantly larger in Year 5 compared with Year 3, and significantly larger again in Year 7 compared with Years 3 and 5. For example, in 2006, the figure was 7% in Year 3, increasing to 9.7% by Year 5 and 20.3% by Year 7. Overall there was a significant proportion of students not meeting the national numeracy benchmarks, and the situation seems to have deteriorated in the later years of schooling.

After a 10-year period, the *Melbourne Declaration on educational goals for young Australians* (MCEETYA, 2008), which replaced the *Adelaide Declaration* (1999), established guidelines for Australian schooling for the ten-year period, 2009–18. The *Melbourne Declaration,* endorsed by all Australian State and Territory Ministers of Education, has two most important educational goals for young Australians: first that Australian schooling should promote equity and excellence; and second that all young Australians should become successful learners, confident and creative individuals, and active and informed citizens (MCEETYA, 2008, p. 8–9). The *Melbourne Declaration* continues to identify literacy and numeracy as the cornerstones of schooling. As described further in Goal 2, successful learners' attributes should include having the "essential skills in literacy and numeracy" (MCEETYA, 2008, p. 8).

Students' performance in numeracy over the six-year period (2008–13) in relation to the percentage of students not meeting the national numeracy standard is now reviewed.

Table 1.2 Percentage of Years 3, 5 and 7 students not meeting the national numeracy benchmarks in Australia, 2008–13

Year	Year 3	Year 5	Year 7
2008	5.0	7.3	4.6
2009	6.0	5.8	5.2
2010	5.7	6.3	4.9
2011	4.4	5.6	5.5
2012	6.1	6.7	6.2
2013	4.3	6.6	5.0

Source: Extracted from ACARA, 2013, p. 279.

Table 1.2 indicates that numeracy achievement at the national level in Years 3, 5 and 7 has fluctuated and there is very little change over the six-year period, 2008–13. There was a slight improvement in numeracy achievement in Year 3 over the period 2008–13 compared with the previous period 2000–06. As well, there was a remarkable improvement in Years 5 and 7 from 2000–06 to 2008–13. For example, in 2006 there was 9.7% of Year 5 students not satisfying the national numeracy benchmarks. By 2008, this figure had decreased to 7.3% and remained steady in the following years in the period under review.

Although the review of numeracy achievement shows some improvement over the last 15 years, and the goal that every child should meet a minimum acceptable numeracy standard has been set in the two national statements (the *Adelaide Declaration*, 1999 and the *Melbourne Declaration*, 2008), the proportion of students not meeting the national numeracy benchmarks needs considerable improvement. Thus there is a significant need for teaching interventions for those students.

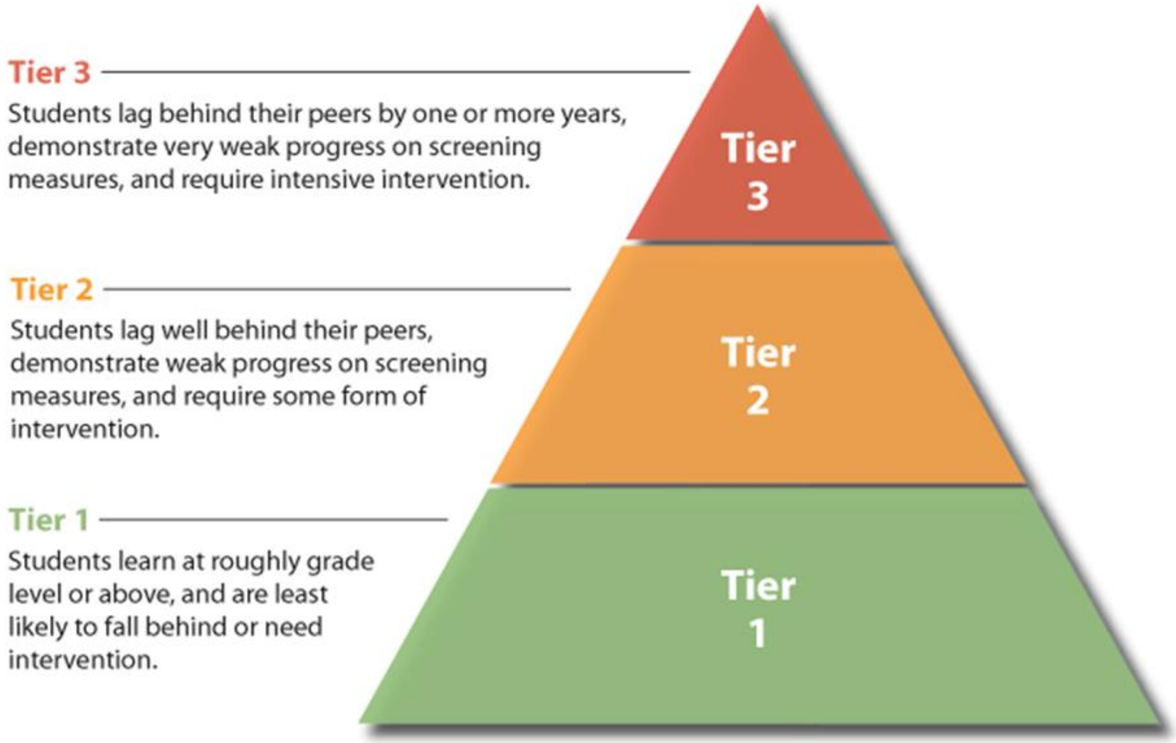
1.1.3 Responses to the Identification of Mathematical Learning Difficulties

Research in mathematics education has indicated that early intervention can be significant for students with mathematical learning difficulties. Students begin school with different levels of mathematical knowledge (Aubrey, 1993; Baroody, 1987; Wright, 1991, 1994a; Wright, Ellemor-Collins, & Lewis, 2007; Young-Loveridge, 1989). Students who are low-attaining in the early years tend to remain so in their mathematics achievement throughout their schooling (Aubrey, Dahl, & Godfrey, 2006; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Wright, Martland, & Stafford, 2006). Research also has indicated that without intervention, this initial gap in mathematics achievement keeps increasing during secondary schooling (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). Therefore, it seems necessarily to provide low-attaining students with the opportunity for an educational intervention at early stage before they fall far too behind compared with the group of average and high-attaining students, as Wright, et al. (2006,) have observed. Hence, it is important that such intervention in students' learning of mathematics should occur in their early school years.

In Australia, there have been many attempts to support students who do not meet the national numeracy standard. This work has resulted in several intervention programs designed to address learning difficulties in mathematics. These programs include supporting low-attaining students within a classroom, providing instruction in small groups of two or three students, and providing individualized one-to-one intervention programs.

A review of numeracy interventions in the early years of schooling in Australia (Meiers, Reid, McKenzie, & Mellor, 2013) involved analysing the research evidence for the efficacy and effectiveness of numeracy interventions. These interventions were grouped in line with the tiered structure of a Response to Intervention (RtI) framework consisting of three tiers of support (see Figure 1.1). Figure 1.1 describes the three tiers of response to intervention that presented as a kind of pyramid for supporting students with learning difficulties. The amount of support that a student receives increase with each level. The interventions reviewed were classified as either Tier 1 or Tier 2. None was designed specifically as a Tier 3 intervention (Meiers et al., 2013).

Figure 1.1 Three tiers of support



Source: ALEKS, 2016, p. 1

Meiers et al. (2013) list 18 numeracy interventions (see Table 1.3), which were best described as Tier 1 or 2 in the RtI framework (Fuchs & Fuchs, 2006). These interventions have been used in Government, Catholic and Independent schools in Australia. The name, origin and source of each intervention program appear in Appendix 1. Table 1.3 provides an overview of numeracy interventions in Australia.

Table 1.3 Overview of the numeracy interventions in Australia

Australian Numeracy Interventions	
Tier 1	**Origin**
Count Me in Too (CMIT)	NSW
Count Me in Too Indigenous (CMITT)	NSW
First Steps in Mathematics	Western Australia
Learning in Early Numeracy (LIEN)	NSW
Mathematics in Indigenous Contexts Project	NSW
Numeracy Matters	NSW
Success in Numeracy Education (SINE)	Victoria
Taking off with Numeracy (TOWN)*	NSW
Tier 2	**Origin**
Best Start Targeted Early Numeracy (TEN)	NSW
Extending Mathematical Understanding Intervention (EMU)	Victoria
Getting ready in Numeracy (GRIN)	Victoria
Mathematics Intervention	Victoria
Mathematics Recovery (MR)	NSW
Numeracy Intervention Project (NIP)	NSW
Numeracy Intervention Research Project (NIRP)	Victoria
QuickSmart Numeracy	NSW
Taking off with Numeracy (TOWN)	NSW
Train a Math Tutor Program	Queensland

Source: Meiers, Reid, McKenzie, & Mellor, 2013, p. 67.

Note: TOWN is both a Tier 1 and Tier 2 numeracy intervention program.

Meiers et al. (2013) found the lack of research evidence available for the efficacy and effectiveness of the numeracy interventions reviewed. In addition, there is little research evidence underlying the efficacy and effectiveness of specific intervention to enhance students' mathematical performance at early stage of schooling, that is, Years K-3. Given the lack of systematic studies of the cost-effectiveness of numeracy interventions (Meiers et al., 2013, p. xii), it is challenging to reach a conclusion about the efficacy and effectiveness of intervention programs.

Meiers et al., (2013, p. 68) suggested that:

> It should be emphasised that the literature review focused on the strength of the evidence for specific numeracy interventions. A lack of evidence for an intervention does not necessarily indicate that the intervention is ineffective; instead, it may indicate the need to collect more rigorous data to evaluate whether the intervention achieves its intended aims.

This present investigation draws on aspects of the Mathematics Intervention Specialist Program (Wright et al., 2011) and focuses on the Key Elements of intensive intervention teaching. The Mathematics Intervention Specialist Program is described in detail in the following section.

1.1.3.1 Mathematics Intervention Specialist Program

An ongoing eight-year (2009–16) project, the Mathematics Intervention Specialist Program, currently operating in a large school system in Victoria, can be regarded as one important response to the need to provide specialised programs for students having difficulties in learning mathematics. The Mathematics Intervention Specialist Program builds on foundations that include the Mathematics Recovery (MR) program (Wright, 2003, 2008), an intensive one-to-one intervention program in number learning for low-attaining students in Year 1 and the Numeracy Intervention Research Project (NIRP) (Ellemor-Collins & Wright, 2009, 2011a; Wright et al., 2007), which focuses on intervention in the number learning of low-attaining Years 3 and 4 students.

Mathematics Intervention Specialist Program has a distinctive and successful approach to addressing mathematics learning difficulties (Willey, Holliday, & Martland, 2007) based on one-to-one instructional intervention which is intensive, highly interactive and targeted at the student's zone of proximal development (Vygotsky, 1978). According to Vygotsky, the student's zone of proximal development is the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers. Mathematics Intervention Specialist Program uses videotaping of lessons and interview-based assessments for subsequent analysis, and has an intensive, year-long program of professional development for teachers to become specialists in mathematics intervention. The Mathematics Intervention Specialist Program corpus of videoed sessions has been accessed for this present investigation, enabling a focus on expert tutoring in one-to-one instruction.

1.2 Research Problem

Although there has been strong advocacy for individualised programs, there is still a limited body of empirical literature that focuses on teacher-student interactions and teaching practices in one-to-one instruction. Individualised one-to-one intervention programs have come to the forefront recently because of a renewed focus on students being in danger of school failure, together with a renewed governmental commitment of providing all students with basic mathematical knowledge. A review of numeracy interventions in the early years of schooling in Australia (Meiers et al., 2013) indicates that there is a need to strengthen the evidence for specific numeracy interventions. This could involve collecting more rigorous data to enable their efficacy and effectiveness in terms of improving students' mathematical performance to be properly evaluated.

The effectiveness of one-to-one teaching has been well documented in English speaking countries (Bloom, 1984; Cohen, Kulik, & Kulik, 1982; Dowker, 2004; Fantuzzo, King, & Heller, 1992; Graesser, Person, & Magliano, 1995; Slavin, 1987). In addition, research on the effectiveness of one-to-one instruction has found that in helping student learning expert tutors perform more effectively than non-expert ones (e.g., Bloom, 1984; Chae, Kim, & Glass, 2005; Di Eugenio, Kershaw, Lu, Corrigan-Halpern, & Ohlsson, 2006). However, the effectiveness of expert tutors is largely unexplored (Lu, Eugenio, Kershaw, Ohlsson, & Corrigan-Halpern, 2007), and empirical research involving systematic studies of learning gains from the use of expert tutors is limited. Therefore, there is a need to explore in depth the nature of the tutoring strategies that expert tutors implement during highly interactive one-to-one instruction with their students, and which, in turn, might provide additional evidence about the efficacy and effectiveness of a particular numeracy intervention program in terms of improving students' mathematical performance.

Decades of research on one-to-one instruction or tutoring have shown that tutor-student interactions are complex, and a common set of expert tutoring strategies has not yet emerged (e.g., Graesser et al., 1995; Lu et al., 2007; McMahon, 1998; Person, Lehman, & Ozbun, 2007; Wright, 2010; Wright, Martland, Stafford, & Stanger, 2002). Further, the terms used to refer to tutoring strategies used by expert tutors when interacting with their students vary from study to study. These terms include: characteristics of one-to-one teaching, such as those identified by McMahon (1998); the key elements, identified by Wright (2010), and Wright, et al. (2002); the various tutor moves, outlined by Lu et al. (2007); and dialogue moves, described by Person et al. (2007). Thus, it is apparent that tutoring research has not converged on a widely agreed-upon theory of how tutoring strategies should be segmented and characterized, as Ohlsson et al. (2007, p. 350) argue.

In the present investigation, the one-to-one instruction was conducted by the teachers who have undertaken the Mathematics Intervention Specialist Program. Those teachers could conceivably be referred to as expert tutors. From here on in this investigation, I refer them as teachers.

1.2.1 The Investigation's Focus and Research Questions

The investigation focuses on elucidating the tutoring strategies that expert tutors implement during highly interactive one-to-one instruction with a student in a particular individualised intervention program—the Mathematics Intervention Specialist Program. The investigation involves identifying and illuminating instructional strategies relating to how the teachers act in particular situations to achieve particular pedagogical goals. A set of instances of such instructional strategies, called Key Elements of one-to-one instruction, is conceptualised. The definition of a Key Element is as follows.

A Key Element of one-to-one instruction is a micro-instructional strategy used by a teacher when interacting with a student in solving an arithmetical task. It is considered to be the smallest unit of analysis of teaching, as portrayed in Figure 1.2, and it has at least one of four functions, as described in Figure 1.3.

Figure 1.2 A hierarchy of events occurring in one-to-one instruction

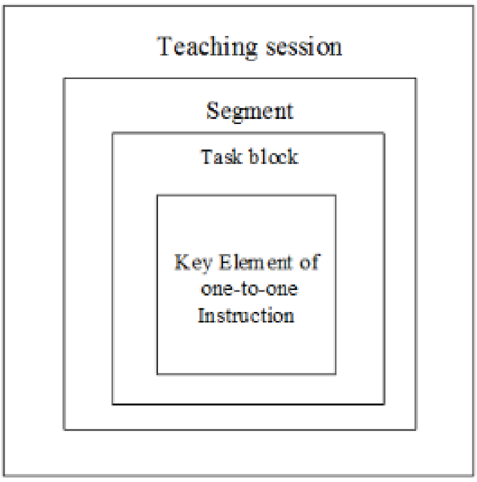


Figure 1.2 shows a hierarchy of events in one-to-one instruction. The uppermost level consists of a one-to-one teaching session and the lowest consists of Key Elements. A mathematics intervention teaching session is a structured period of instruction and typically is of approximately 30 minutes in duration. A segment is a part of a mathematics intervention teaching session where a teacher uses a particular setting or a collection of settings for

instruction in a particular learning domain. The term “setting” refers to a situation used by the teacher when posing an arithmetical task. A setting consists of physical materials and written or verbal statements, and the ways in which these are used in instruction and feature in a student’s reasoning. A *task block* is a part of a segment which starts at the point where a teacher poses a task and ends when the student solves the task.

Figure 1.3 Functions of Key Elements of instruction

Organising on-task activity The process by which a teacher chooses teaching materials and organises a physical setting in a teaching session, with intent to engage a student in interactions in order to facilitate student learning.
Responding to student thinking or answering The process by which a teacher’s subsequent instructional choices are in response to a student’s strategies or thinking. The teacher’s instructional choices include providing support where necessary, explaining mathematical aspects relevant to the current instruction, and promoting discussion in response to the student’s explanations of their attempts to solve a task.
Adjusting task challenge within a task The process by which the teacher reduces the level of difficulty of a task when the student cannot answer correctly, particularly in cases where the student does not seem to be using a strategy.
Providing opportunities for students to gain intrinsic satisfaction from solving a task The process by which a teacher inspires a student in ways that lead naturally to the student (a) experiencing success with challenging tasks; (b) checking and confirming solutions; and (c) being aware of and celebrating progress.

A major focus of this investigation was to identify and illuminate the essence (Van Manen, 1997, p. xiv) of Key Elements of one-to-one instruction in whole-number arithmetic. This involved conceptualising a framework of Key Elements for analysing one-to-one instruction in whole-number arithmetic. To achieve the purposes described above, the following research questions were addressed:

1. What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?
2. How can the Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

1.3 Conceptual Framework

The present investigation was concerned with instructional strategies that expert tutors use during highly interactive one-to-one intervention instruction. The conceptual framework of the present investigation draws on constructivism in mathematics education and inquiry-based instruction in mathematics. Constructivism has been applied to various areas of mathematics education. In the present investigation, it is applied to situations involving expert tutoring focused on low-attaining 3rd and 4th graders' learning of whole-number arithmetic. A constructivist view of learning can constitute a lens for examining learning that occurs in instructional situations that are highly interactive and focused on the learning of arithmetic. The constructivist perspective on learning and inquiry-based instruction of mathematics will be reviewed in detail in chapter 2.

The present investigation focuses particularly on analysing one-to-one instruction in Mathematics Recovery, which includes taking account of nine guiding principles of mathematics intervention teaching and nine characteristics of children's problem-solving (Wright, Martland, Stafford, & Stanger, 2006, pp. 25-31, 37-40). The framework involving the nine guiding principles and nine characteristics, therefore, essentially informs how to define a Key Element of one-to-one instruction and how to identify Key Elements in the data analysis phases. Table 1.4 lists these principles and characteristics. A more detailed description of the guiding principles and characteristics is provided in Chapter 2.

Table 1.4 Guiding principles of Mathematics Recovery teaching and characteristics of children's problem-solving

No.	The nine guiding principles of Mathematics Recovery teaching
1	Inquiry-based teaching
2	Initial and ongoing assessment
3	Teaching just beyond the cutting edge (ZPD)
4	Selecting from a bank of teaching procedures
5	Engendering more sophisticated strategies
6	Observing the child and fine-tuning teaching
7	Incorporating symbolizing and notating
8	Sustained thinking and reflection
9	Child intrinsic satisfaction
	The nine characteristics of children's problem-solving
1	Cognitive reorganization
2	Anticipation
3	Curtailment
4	Re-presentation
5	Spontaneity, robustness and certitude
6	Asserting autonomy
7	Child engagement
8	Child reflection
9	Enjoying the challenge of problem-solving

Source: Wright, Martland, Stafford, & Stanger, 2006, p. 26.

1.4 Significance of the Investigation

This investigation addresses the issue of student difficulties in learning mathematics in primary schools, which is a significant problem in Australia as well as in the United States, the United Kingdom, New Zealand and elsewhere (see, for example, Ellis, 2005). In Australia, for example, economic and social development may be significantly enhanced if greater numbers of students learn mathematics successfully at schools (ACARA, 2013, p. 8; MCEETYA, 1999; National Numeracy Review Report, 2008).

Investigating the mathematical intervention of intensive, one-to-one teaching of Years 3 and 4 students with learning difficulties is a logical progression from previous studies on Year 1 students such as that conducted by Wright, et al. (2002). It provides the potential to review and extend the existing framework developed by investigating Mathematics Recovery intervention teaching. Such a review and extension are justified given the qualitative differences in the teaching for these two groups, which may result in different Key Elements arising during instruction. The differences may be partly due to the arithmetic content for intervention students at Years 3 and 4, which is significantly different from that at Year 1. Intervention in Year 1 focuses on topics such as identifying numerals, saying sequences of number words and counting items that are seen, whereas by Years 3 and 4 the focus is on place value, and addition and subtraction involving numbers beyond 10. Additionally, in the case of Years 3 and 4 students, there is a much greater emphasis on formal recording methods when working out problems, such as the writing of number sentences, for example, 35 + 28 = 63.

The role and nature of teacher-student interactions in developing students' conceptual understanding and mathematical knowledge construction has been emphasised recently (Grandi & Rowland, 2013). Further, teaching practice which builds on students' mathematical thinking to develop mathematical concepts is valued by the mathematics education community (e.g., Leatham, Peterson, Stockero, & Van Zoest, 2015; Lester, 2007). The present investigation addresses the need to further develop the set of comprehensive descriptions of Key Elements. The development of a more comprehensive and robust set of descriptions of Key Elements would enable a deeper understanding of teacher-student interactions and teaching practices in one-to-one instruction. This in turn, allows an extension and refinement of the relevant research literature that informs teaching approaches designed to build on students' mathematical thinking.

One potential outcome of this investigation is expected to be a more comprehensive framework for analysing intensive, one-to-one instruction. This framework potentially might enhance our understanding of how a teacher uses a specific cluster of Key Elements to achieve particular pedagogical goals. The framework may also serve as a guide to the ways in which teachers interact with low-attaining students, providing useful information about effective teacher strategies in mathematical intervention, which in turn may impact positively on the outcomes of student learning. Additionally, the methods that teachers use, or learn to use, in one-to-one intervention teaching, can be applied potentially across the whole range of student attainment and classroom contexts (Tran & Wright, 2014a). A more comprehensive framework may also be useful to instructional leaders and administrators for analysing and evaluating the effectiveness and appropriateness of one-to-one intervention teaching, which, in turn, might benefit schools and education systems because the resulting framework should be able to be applied to strengthening classroom instruction as well as intervention instruction (Tran & Wright, 2014a).

It is expected that the investigation will shed light on why a focus on the Key Elements of one-to-one instruction might result in successful learning outcomes. It is anticipated that these findings will give rise to an extension and refinement of the empirical literature relevant to intensive intervention in number and early arithmetical learning and to the further development of a more advanced, more comprehensive theoretical framework.

1.5 Methodology

The theoretical perspective underpinning the present investigation is interpretative. Within this perspective, a *phenomenological approach* is used to develop comprehensive descriptions of the Key Elements using analytical techniques described by Van Manen (1990, 1997). A qualitative methodology is employed to gain insight into the nature of the Key Elements in the context of intensive, one-to-one intervention teaching. In the present investigation, Key Elements are viewed as the central phenomena requiring exploration and deeper understanding. The approach adopted, therefore, is essentially phenomenological and serves to describe the essence of Key Elements. The qualitative research methods of systematic observation and phenomenological analysis are followed to ensure the rigour of the chosen methodology.

The primary data source for the present investigation is drawn from the Mathematics Intervention Specialist Program in which teachers provided intensive, one-to-one instruction to low-attaining Year 3 and Year 4 students (Wright et al., 2011). The participants consisted of four teachers and six students. Two teachers each taught two students singly and the other two each taught one. The four teachers were selected from a pool of approximately 50 teachers in the Mathematics Intervention Specialist Program and were regarded by Mathematics Intervention Specialist Program leaders as being particularly competent in intervention teaching. Thus, the data involves six sets of video recordings of teaching sessions. Each set consists of up to eight teaching sessions, each of 30-45 minutes' duration. This results in approximately 33 hours of video for analysis.

A methodological approach for analysing large sets of video-recordings (Cobb & Whitenack, 1996) and a model for the analysis of video data (Powell et al., 2003) were adopted in this investigation. The videos were transcribed and then coded with respect to the Key Elements of one-to-one instruction by using the NVivo 10 software program (QSR International Pty Ltd. Version 10, 2012).

1.6 Organisation of the Investigation

The thesis is organised as follows. Chapter 1, *Introduction*, provides the background, context and rationale for the investigation. The chapter also describes briefly the conceptual framework, research problem, aim and methodology for the investigation. It also outlines the associated research questions.

The purpose of Chapter 2, *Conceptual Framework and Literature Review*, is threefold. The first part provides a detailed explanation of the conceptual framework. The second part situates the present investigation in the broader context of educational and mathematical literature involving perspectives on effective teaching practices, and, in particular, those addressing learning difficulties in mathematics. The final part explains the key findings of existing research on one-to-one instruction in order to identify relationships among the key findings and any gaps in the research.

Chapter 3, *Methodology*, provides a detailed account of the research methodology. The chapter elaborates phenomenology as an appropriate methodology for an investigation in which Key Elements are viewed as the central phenomenon to be further explored and better understood. Key methodological considerations and the criteria for trustworthiness are explained in full. Finally, the processes of observation and data analysis are described.

Chapter 4, *Analysis of Data*, reports on how the processes of observation and data analysis have been carried out in the present investigation. First, the data set, and in particular, the use of the video recordings, is explained. Second, the analytical framework for investigating Key Elements, which evolved during the analysis process, is explained in detail.

Chapter 5, *Key Elements of Intensive, One-to-one Instruction*, focuses on answering the first Research Question by identifying and illuminating the Key Elements used in intensive, one-to-one intervention teaching. This chapter provides comprehensive descriptions of the Key Elements identified and provides excerpts from Mathematics Intervention Specialist Program teaching sessions in order to illustrate them. Problematic teacher behaviours associated with one-to-one instruction, as identified during the data analysis phase, are also explained as an additional outcome of the investigation.

Chapter 6, *A Framework for Analysing Intensive, One-to-one Instruction*, focuses on answering the second Research Question. Accordingly, this chapter provides a comprehensive framework for analysing one-to-one instruction. It identifies the necessary contextual elements for understanding how a teacher might implement a specific cluster of Key Elements in order to achieve particular learning outcomes.

Chapter 7, *Discussion*, focuses on interpreting the findings, discussing them in relation to the literature, and explaining the implications of the findings. It includes a brief summary of key aspects of the findings and then explains the value of the findings to the knowledge base. Finally, the chapter provides discussions on the findings in light of previous research in the field.

Chapter 8, *Conclusion*, synthesises the empirical findings of the investigation with respect to the research questions. It discusses the theoretical and methodological contributions that the investigation makes to the field, and it explores the implications of the investigation for policy, and potentially for mathematics intervention programs, especially one-to-one intervention in primary schools. As well, the chapter acknowledges the limitations of the investigation and provides some suggestions for future research.

Chapter 2 – **Conceptual Framework and Literature Review**

The focus of this chapter is threefold. The first is to describe a conceptual framework informed by concepts, beliefs, and teaching and learning theories that support and inform this investigation. The second is to situate the present investigation in the broader educational and mathematical literature. The third is to describe key findings of research on one-to-one instruction and to identify relationships among the key findings and any gaps in the research. Through discussion of these three foci, the theoretical perspective underlying the study is detailed. The reason for choosing this structure is to present a combined theoretical perspective and literature review. In this way the literature review is informed by and will inform the theoretical perspective. This approach enables a rich engagement of the literature review with the theoretical perspective.

2.1 Conceptual Framework

A conceptual framework presenting concepts, beliefs and theories that support and inform one's research is a significant part of the design of an investigation (Robson, 2011). Miles and Huberman (1994, p. 18) describe a conceptual framework as a visual or written product that "explains, either graphically or in narrative form, the main things to be studied – the key factors, concepts, or variables – and the presumed relationship between them".

The present investigation is concerned with instructional strategies that expert tutors use during interactive one-to-one intervention teaching. These strategies are referred to as Key Elements of one-to-one instruction. The conceptual framework for the investigation draws on constructivism in mathematics education, on inquiry-based instruction in mathematics, and, in particular, on the nine guiding principles of Mathematics Recovery teaching and the nine characteristics of children's problem-solving (Wright, Martland, Stafford, et al., 2006, pp. 25-31, 37-40). Teacher professional noticing (Jacobs, Lamb, & Philipp, 2010) and dimensions of mathematisation (Ellemor-Collins & Wright, 2011b) serve also as a lens for unpacking the in-the-moment decision making associated with the use of Key Elements.

Constructivism has been applied to various areas of mathematics education. In the present investigation, constructivism is applied to situations involving expert tutoring focused on low-attaining 3rd and 4th graders' learning of whole-number arithmetic. A constructivist view of learning can constitute a lens for examining learning that occurs in instructional situations that are highly interactive and focused on the learning of arithmetic. This section begins with an articulation of the constructivist perspective that underpins the study; it then provides a review of the pedagogical theory based on constructivism that preceded this study and contributed to its theoretical foundation.

2.1.1 A Constructivist Perspective on Learning

This section describes a constructivist perspective on learning, based on the empirical and theoretical work of Piaget (e.g., Piaget, 1964) and of theorists, educators and researchers within mathematics education (e.g., Cobb, Yackel, & Wood, 1992; Steffe & Cobb, 1988; Von Glasersfeld, 1983). According to Cobb (2000, p. 277):

> A range of psychological theories about learning and understanding fall under the heading of constructivism. The common element that ties together this family of theories is the assumption that people actively build or construct their knowledge of the world and of each other. This claim applies as much to perception as it does to higher-order reasoning and problem solving. Consequently, constructivists reject the view that people's perceptual experiences are direct, unmediated responses to stimuli, and instead argue that perception involves processes of interpretation that may be very abbreviated in routine instances of recognition. In addition, constructivists question the view that remembering involves the direct retrieval of information stored in memory. They instead contend that remembering is a reconstructive process in which we recall past incidents and events in terms of current understandings.

The term, 'constructivism', has been used in education and educational psychology with increasing frequency since the late 1970s. Currently, any serious discussion of learning theory related to mathematics, science or literacy would include a discussion of constructivism. Researchers and writers have used the term with differing meanings and in different contexts. This situation has led to the emergence of a range of sub-fields of constructivism, such as cognitive constructivism (e.g., Piaget, 1977), radical constructivism (Steffe & Thompson, 2000b; Von Glasersfeld, 1995), and social constructivism (e.g., Lerman, 2000). As well, constructivism is used to refer not only to student learning but also to teaching (e.g., Steffe & Gale, 1995). The following section provides a description of the particular constructivist perspective that underlies the present investigation.

Since the 1970s, the term 'radical constructivism', as coined by Von Glasersfeld (1978, 1991, 1995), has played a significant role in research and writing about mathematics education. Radical constructivism is a theory of knowing. According to Von Glasersfeld (1995, p. 1), radical constructivism is:

> an unconventional approach to the problems of knowledge and knowing. It starts from the assumption that knowledge, no matter how it be defined, is in the heads of persons, and that the

> thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. What we make of experience constitutes the only world we consciously live in. it can be sorted into many kinds, such as things, self, others, and so on. But all kinds of experience are essentially subjective, and though I may find reasons to believe that my experience may not be unlike yours, I have no way of knowing that it is the same.

Given that radical constructivism builds on Piaget's theory of genetic epistemology (Piaget & Duckworth, 1970), some of the key ideas in radical constructivism are ideas developed by Piaget. These include mental processes such as assimilation, accommodation and equilibration, and psychological notions such as schemes, mental reflection and re-presentation.

Radical constructivism is not a theory of learning but it is very useful as a foundation for the development of theories about how particular aspects of mathematics are learned. Steffe, for example, has drawn on radical constructivism in developing extensive theories about students' learning of aspects of early number and arithmetic (Steffe, 1994; Steffe & Cobb, 1988). Those who study learning from a radical constructivist perspective take very seriously the importance of trying to describe the student's current strategies and knowledge, that is, to understand mathematics from the student's perspective. Understanding mathematics from the student's perspective is regarded as a key first step in attuning instruction to students' current ways of thinking. Related to this is the view that instruction that is closely attuned to students' current ways of thinking is more likely to be successful. In the theory of radical constructivism, understanding mathematics from the student's perspective entails the process of conceptual analysis, that is, determining the mental operations that would result in seeing this mathematical situation in the way a student sees it (Von Glasersfeld, 1995, p. 78).

Radical constructivists consider that knowledge development should primarily be seen as a cognitive process because it focuses on the individual's construction. Thus radical constructivists take a cognitive, or psychological, perspective. In contrast, epistemologists adopting a sociocultural perspective see knowledge development as being primarily a social process. According to Wertsch and Toma (1995, p. 160), for example, "Sociocultural processes are given analytical priority when understanding individual mental functioning, rather than the other way around". From a sociocultural perspective, learning cannot happen if the individual stays apart from interacting either with other individuals or with cultural artifacts. This approach appears to be consistent with the work of Vygotsky (1978, p. 57):

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the

> formation of concepts. All the higher functions originate as actual relations between human individuals.

Vygotskian theory is regarded as being particularly relevant to understanding the role of instruction in learning. According to Vygotskian theory, instruction should lead development, rather than follow it. A key notion in the application of Vygotskian theory to education is that of 'the zone of proximal development' (Vygotsky, 1978, pp. 84–91). This is the zone of learning that is within the student's grasp, with the assistance of teaching, but the student is unlikely to learn without assistance. Vygotskian and sociocultural theory have been applied widely in education and have been used in mathematics education to understand better the nature of collaborative learning in classrooms (e.g., Cobb & Bauersfeld, 1995) and the role of symbolising and communication in mathematics learning (Cobb, Stephan, McClain, & Gravemeijer, 2001).

The theoretical position of the present investigation draws on elements of both radical constructivism and a sociocultural perspective. It draws on the theoretical work of Cobb, Yackel, and Wood (Cobb, 1989; Cobb & Bauersfeld, 1995; Cobb, Wood, & Yackel, 1993; Cobb & Yackel, 1996), and of Bauersfeld (1995), whose theories are grounded in both radical constructivism (Von Glasersfeld, 1991) and symbolic interactionism (Blumer, 1969), a sociocultural perspective. The theoretical position of the present investigation is, therefore, quite close to an emergent perspective (Cobb, 1996; Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997) in which radical constructivism, which is used to analyse students' mathematical thinking and strategies, is coordinated with symbolic interactionism, which is used to analyse teacher-student interactions and discourse.

While most of the works referred to above are based on learning in classroom contexts, the present investigation focuses on learning in a one-to-one context. It is important to clarify the constructivist perspective of learning in one-to-one contexts on which the present investigation is based. Scenario 2.1 (see Figure 2.1) presented below exemplifies what, in the present investigation, is meant by a student actively constructing arithmetical knowledge in a one-to-one instruction context.

Figure 2.1 Scenario 2.1 Sophia – Ben

Scenario
Sophia: (Looks at Ben) Want to have a go at near doubles?
Ben: Mm hmm.
Sophia: I reckon you can do it. (Brings out an arithmetic rack in front of Ben) So, a five- (counts beads). So, if I give you a sum, say, like this (slides five blue beads from right to left with three on the upper row and two on the lower row), what's this sum?
Ben: Three and two is, is four … five.
Sophia: Five, good. How do you know that's five?
Ben: Because three (indicates the three blue beads on the upper row) and I add one more is five, no is four and I add one more is five (points to the two blue beads in turn on the lower row).
Sophia: Mm okay good. Can you see another way you might be able to solve that?
Ben: Mmm, two and two is four and one more is five.
Sophia: Good. Good. Two and two is four and one more is ...
Ben: Five.
Sophia: Five. Good. (Slides the five blue beads back to the right of the rack) What about this one? (Slides seven blue beads from right to left with four on the upper row and three on the lower row)
Ben: Four and three is, four and three is seven.
Sophia: Good. How do you know that?
Ben: Because if you just, if you do, um, five and-, three and three is six and one more is seven. (Points at the beads when talking)
Sophia: Excellent, Ben. I really like that way. Okay. Because you're not really counting by ones then are you. You know when you did that one (makes the 3+2 sum on the rack with three blue beads on the upper row and two blue beads on the lower row), you sort of go three, four, five. (Points at the beads to illustrate what Ben did when first solving the task of 3+2).
Ben: Mm hmm.
Sophia: That's sort of what you do, um, maybe at the start of grade, you know, we really want you to find a different way to do things now rather than counting by ones, okay, now that you're in grade four. So, that's good what you said there. (Makes the 4+3 sum on the rack with four blue beads on the upper row and three blue beads on the lower row). Four and three is...
Ben: Four-
Sophia: Three and three ... (Points at the three beads on the upper row and the three beads on the lower row at the same time)
Ben: Three and three is six and add one more is ...
Sophia: That's a, that's a really good way to do it.

The scenario involves the teacher, Sophia, and her student, Ben. It focuses on 'near doubles', for example, 4+3 which is considered to be near the double of 3. Sophia uses the setting of an arithmetic rack in presenting the tasks. In the first task, 3 +2, Ben's arithmetical strategies are typically based on counting by ones. He solves the task 3+2 by counting-on two counts from 3, '3: 4, 5'. Sophia expected Ben to develop facile strategies for solving addition tasks in the range 1 to 10. Thus she asks Ben to solve the task another way. Ben then came up with a more sophisticated strategy which does not involve counting by ones. Rather, it involves partitioning 3 into 2-and-1, reorganising 3+2 as 2+2+1, and knowing the combination 2+2 without counting. In the next task, 4+3, Ben gradually becomes accustomed to the strategy of using a double to solve tasks. He can see 4+3 as 3+3+1 and he uses the double of 3 without counting by ones.

Throughout the scenario, Ben shows that he is able to actively construct knowledge. This involves using his prior knowledge of doubles and addition, and results in a strategy that is more sophisticated than his earlier strategy. This kind of thinking is referred to as *additive structuring of numbers* (Ellemor-Collins & Wright, 2009, p. 53) because it involves regarding numbers as consisting of parts that are reorganised to facilitate solving a task. A progression in students' arithmetical thinking from their calculating being based solely or mainly on counting by ones to being based on structuring numbers is a crucial development (Steffe & Cobb, 1988; Wright, Ellemor-Collins, & Tabor, 2012; Young-Loveridge, 2002).

2.1.2 Mathematical Instruction: Theoretical Considerations

2.1.2.1 Inquiry-based Instruction of Mathematics

According to Simon (1995, p. 114), although constructivism provides a useful tradition for mathematics educators to understand mathematics learning, the mission of developing a teaching approach on the basis of a constructivist view of learning is challenging. Inquiry-based instruction is a pedagogical approach based on such a view. This approach was developed during the *discovery learning* movement of the 1960s (e.g., Bruner, 1961) as a response to traditional forms of instruction that focused on rote learning and memorisation. Inquiry-based instruction is a process that involves posing questions or problems about a specific topic. This process is generally assisted by the teacher in the role of a facilitator.

The present investigation involves analysing data resulting from the Mathematics Intervention Specialist Program. In the analysis, the researcher served as a 'complete observer' (Cohen, Manion, Morrison, & Morrison, 2011, p. 457), that is, the researcher's role was separate from the participants. Indeed, the present investigation is a retrospective study that involved the researcher analysing video recordings of teaching sessions already conducted in the Mathematics Intervention Specialist Program. Teachers in the Mathematics Intervention Specialist Program had undertaken specialist training focused mainly on inquiry-based instruction.

From a constructivist perspective, the inquiry-based approach to instruction provides a lens for the researcher through which to analyse the teaching as it occurs in the data, that is, the video recordings. Within inquiry-based instruction, students' learning wholly or mainly involves students solving arithmetical problems that are challenging but at which they have a good chance of being successful. To the casual onlooker, these problems seem to be simple and perhaps mundane to some extent. Nevertheless, when the intervention teacher has an elaborated view of the student's current knowledge, an arithmetical task can be presented that is likely to engage the student in an episode of significant problem solving.

Furthermore, Wright et al. (2012) argued that students not only need to solve challenging problems to progress in mathematical sophistication, but also they need to rehearse their strategies to develop grounded habituation of knowledge. Therefore, it is worthwhile to distinguish two productive modes of work that students can adopt: inquiry mode and rehearsal mode. The inquiry mode is mainly based on a constructivist perspective of learning, whereas the rehearsal mode is mainly based on a behavioural perspective.

According to Wright et al. (2012), the inquiry mode involves students solving a challenging problem, exploring some new material or generating further examples. This process produces new knowledge for the students and breaks new mathematical ground. On the other hand, the rehearsal mode involves rehearsing something that has been introduced before, for example, identifying numerals, and reciting number word sequences. This mode involves rehearsing arithmetic knowledge with which the student is acquainted, with the intention of increasing familiarity and ease, and perhaps working towards automatisation.

Some advocates of constructivism argue that there is a need a paradigm shift away from instructional approaches associated with behavioural models to instructional approaches associated with constructivist models. Nevertheless, in the present investigation, these instructional approaches are not regarded as alternatives. Rather, each provides different insights into the nature of learning. As well, the use of an instructional approach in a particular situation depends on what is being taught. In this regard, arithmetic should be considered as subject matter that develops strong integrated mathematical knowledge. If students have their own informal knowledge and use this to invent or construct new knowledge, they may be able to invent solutions and make sense of mathematical problems. On the other hand, learning arithmetic sometimes involves memorising basic facts and mastering procedures or algorithms for the four basic mathematical operations: addition, subtraction, multiplication and division.

Scenario 2.2 (see Figure 2.2) presented below exemplifies what is meant in the present investigation by the inquiry mode occurring in a one-to-one instruction context. The scenario involved the teacher, Amilia, and her student, Mia. It focused on 2-digit addition without regrouping. Amilia used the setting of bundling sticks in presenting the task, 34+21. In the first five seconds after the task was posed, Mia seems to lose track of the first-mentioned addend. Fifteen seconds after the task was re-posed, Mia responds with the correct answer, fifty-five. Amilia asked Mia to check the answer and explain her strategy in solving the task. This scenario provided an example of what in the present investigation is regarded as inquiry-based instruction.

Figure 2.2 Scenario 2.2 Amilia – Mia

Scenario
Amilia: Okay. Let’s start. (Places out three bundles and 4 sticks on the top row, and then places out two bundles and 1 stick on the lower row on the table). If I have got thirty-four (places out three bundles and 4 sticks on the table) and I want to add on twenty-one (places out two bundle sticks and 1 stick in a row under the previous bundles and sticks, and then screens all the bundles and sticks with a black cover), how many do I have altogether? (Looks at Mia) Mia: … (After 5 seconds) Wait. What was the top one again? Amilia: (Unscreens the sticks and points at 34 and 21 sticks in turn) Thirty-four and twenty-one. Mia: … (After 15 seconds) Fifty five. Amilia: Okay. Let’s check. (Brings out the workbook and writes down the sum 34+21=55). So we said thirty-four plus twenty-one and you said fifty-five. Check it with the sticks. (Unscreens the sticks) Mia: Because that’s fifty and that’s five (moves the two bundles of the 21-pile together with the other 3 bundles of 34-pile, and puts the one stick of 21-pile together with the other 4 sticks of 34-pile). Amilia: Beautiful.

2.1.2.2 Guiding Principles of Mathematics Recovery Teaching and Characteristics of Children's Problem-solving

The present investigation focuses on identifying and illuminating Key Elements of one-to-one instruction in Mathematics Recovery, and in particular in the Mathematics Intervention Specialist Program, which are embedded in nine guiding principles of mathematics intervention teaching and nine characteristics of children's problem-solving (Wright, Martland, Stafford, et al., 2006, pp. 25-31, 37-40). Therefore, the framework involving the nine guiding principles and nine characteristics essentially informed how to define a Key Element of one-to-one instruction and how to identify Key Elements in the data analysis phases. Table 2.1 describes briefly each guiding principle of mathematics intervention teaching and the nine characteristics of children's problem-solving.

Table 2.1 Descriptions of the guiding principles of Mathematics Recovery teaching and characteristics of children's problem-solving

Guiding principles of Mathematics Recovery teaching		
No.	**Guiding principles**	**Description**
1	Inquiry-based teaching	The approach to instruction is inquiry based. Students routinely are engaged in thinking hard to solve arithmetical problems which are quite challenging for them.
2	Initial and ongoing assessment	Teaching is informed by an initial, comprehensive assessment and ongoing assessment through teaching. The latter refers to the teacher's informed understanding of the student's current knowledge and problem-solving strategies, and continual revision of this understanding.
3	Teaching just beyond the cutting-edge (ZPD)	Teaching is focused just beyond the 'cutting-edge' of the student's current knowledge.
4	Selecting from a bank of teaching procedures	Teachers exercise their professional judgment in selecting from a bank of teaching procedures each of which involves particular instructional settings and tasks, and varying this selection on the basis of ongoing observations.
5	Engendering more sophisticated strategies	The teacher understands students' arithmetical strategies and deliberately engenders the development of more sophisticated strategies.
6	Observing the child and fine-tuning teaching	Teaching involves intensive, ongoing observation by the teacher and continual micro-adjusting or fine-tuning of teaching on the basis of her or his observation.
7	Incorporating symbolizing and notating	Teaching supports and builds on the student's intuitive, verbally based strategies and these are used as a basis for the development of written forms of arithmetic which accord with the student's verbally based strategies.
8	Sustained thinking and reflection	The teacher provides the student with sufficient time to solve a given problem. Consequently the child is frequently engaged in episodes which involve sustained thinking, reflection on her or his thinking and reflecting on the results of her or his thinking.

9	Child intrinsic satisfaction	Students gain intrinsic satisfaction from their problem-solving, their realization that they are making progress, and from the verification methods they develop.
Characteristics of children's problem-solving		
No.	**Characteristics**	**Description**
1	Cognitive reorganization	refers to a qualitative change in the way the student regards the problem, and generation of a strategy that was previously unavailable to the student.
2	Anticipation	refers to a realization by the student prior to using a strategy that the strategy will lead to a particular result.
3	Curtailment	refers to the mental process of cutting short an aspect of problem-solving activity when, prior to commencing the activity, the student has an awareness of the results of the activity and thus the activity becomes redundant.
4	Re-presentation	refers to a kind of cognitive activity akin to a mental replay, during which the student presents again to herself or himself, a prior cognitive experience
5	Spontaneity, robustness and certitude	A student's strategy is spontaneous when it arises without assistance. A student's strategy is robust when the student is able to use the strategy over a wide range of similar problems. Certitude refers to a student's assuredness about the correctness of their solution to a problem.
6	Asserting autonomy	Students sometimes assert their autonomy as problem-solvers, by imploring the teacher not to help them or to allow them sufficient time to solve a problem independently.
7	Child engagement	The ideal situation in the teaching sessions is for the student to apply herself or himself directly and with effort when presented with a problem, and to remain engaged in solving the problem for a relatively extended period if necessary.
8	Child reflection	refers specifically to a student reflecting on their own prior thinking or the results of their thinking, which can lead to the student becoming explicitly aware of elements of their thinking that were not consciously part of their thinking prior to the period of reflection.
9	Enjoying the challenge of problem-solving	There are many instances in the individualized teaching sessions where students seem to revel in challenging problem-solving, and ultimately they can come to regard problem-solving as an intrinsically satisfying and rewarding experience.

Source: Adapted from Wright, Martland, Stafford, & Stanger, 2006, pp. 25–31, 37–40.

2.1.2.3 Teacher Professional Noticing

The present investigation also links closely to 'teacher professional noticing'—a topic in mathematics education with growing body of research (e.g., Jacobs et al., 2010; Sherin, Jacobs, & Philipp, 2011). Jacobs et al. (2010, p. 169) define teacher professional noticing of students' mathematical thinking as three interconnected skills including (a) attending to students' strategies, (b) interpreting students' understandings; and (c) deciding how to respond to students' understanding and strategies. These skills are described in more detail in Section 7.3.3.2 (Chapter 7).

In the present investigation, teacher professional noticing is considered to be expertise that teachers are required to have in order to use effectively a particular Key Element in a specific instructional situation. Thus, in a sense, it is not about an analysis of what teachers notice, but about what the author as a researcher notices when we study Key Elements in relation to teacher professional noticing.

2.1.2.4 Dimensions of Mathematisation

The growing body of mathematics education research on progressive mathematisation (e.g., Ellemor-Collins & Wright, 2011b; Freudenthal, 1991; Treffers & Beishuizen, 1999) has also influenced the method of data analysis in the present investigation. Wright et al. (2012, pp. 14–15) claim instruction in arithmetic can be viewed in terms of progressive mathematisation, in which advancing students' mathematical sophistication is a crucial task. This approach to instruction is based on students' mathematical thinking and engages with students in problem-solving, visualising, organising, justifying and generalising. Ellemor-Collins and Wright (2011b, pp. 315–317) described ten dimensions of mathematisation in intensive, one-to-one instruction: *complexifying arithmetic; distancing the setting; extending the range; formalising arithmetic; organising and generalising; notating; refining computation strategies; structuring numbers; decimalising numbers;* and *unitising numbers*. In the case of *distancing the setting*, for example, a teacher can progressively distance the setting by instructing a student through steps such as (i) manipulating the physical materials; (ii) seeing the materials but not manipulating them; (iii) seeing them only momentarily; and (iv) solving tasks posed in verbal or written form without materials.

2.1.3 Section 2.1 Concluding Remarks

In Section 2.1, the theoretical position of the present investigation has been presented as involving constructivism, inquiry-based instruction in mathematics and, in particular, the nine guiding principles of Mathematics Recovery teaching and the nine characteristics of children's problem-solving. As well, teacher professional noticing (Jacobs et al., 2010) and dimensions of mathematisation (Ellemor-Collins & Wright, 2011b) serve as theoretical tools for unpacking the in-the-moment decision making associated with using the Key Elements.

2.2 Research on Effective Teaching Practices for Students with Learning Mathematics Difficulties

The present investigation seeks to investigate Key Elements of one-to-one intervention instruction that can be regarded as good teaching practices in a one-to-one context. Thus, the researcher's concern throughout is with the nature of effective teaching practices that address learning difficulties in mathematics; in particular, it is with the kinds of teaching practices in one-to-one, intervention contexts that work for students with difficulties in learning mathematics. This section addresses these concerns and locates the position taken in the present investigation.

2.2.1 Two Theoretical Models of Learning in Mathematics

This section provides an historical overview of general theories of teaching mathematics. This overview addresses a perspective on the major historical controversies that have developed around understanding what effective mathematics teaching and learning are, and the approaches that might be appropriate for students with mathematics learning difficulties.

Ellis (2005, p. 18) described the following forms of learning difficulties:

- performing at low levels across the range of academic tasks;
- having inconsistencies between their cognitive ability and school achievement; and
- being affected by cultural differences, poverty or inconsistent school attendance.

This description indicates that there are various forms or causes relating to student learning difficulties. Therefore, there is no common or sole teaching approach that can be endorsed for all students with learning difficulties (Swanson, 2001, p. 1; Swanson & Deshler, 2003). Certain teaching approaches have been recommended as being particularly effective for students with learning difficulties (Ellis, 2005, pp. 19–20). These approaches are based on two theoretical models of learning. The two models are related to behavioural perspective and a cognitive perspective respectively. Although the research reported here applies to learning difficulties in general, it has also been applied more directly to mathematics learning difficulties.

2.2.1.1 The Behavioural Perspective

Behavioural theory postulates that any particular skill can be enhanced in learning purely through good training and practice (Casey, 1994). Accordingly, this theory suggests that teachers should play an active role in the teaching and learning process (Ellis, 2005, p. 19). Thus, the behavioural perspective supports teachers being authorised to advance students' learning, including students with learning difficulties (Casey, 1994). Adherents of the behavioral perspective have established highly structured instructional approaches that focus on maximising time on tasks and minimising student errors (Ellis, 2005, p. 19). Direct instruction and precision teaching are two popular approaches that are derived from the behavioral perspective. Although the two teaching approaches differ in their presentations, “they both rely on a very structured and carefully monitored system of teaching” (Casey, 1994 cited in Ellis, 2005, p. 19).

The practice of the two behavioural teaching approaches, direct instruction and precision teaching, has been controversial. It is argued that “behavioural approaches are too mechanical and simplistic and that they focus on what might be considered ‘symptoms’ of learning difficulties without addressing the underlying causes” (e.g., Casey, 1994; Farrell, 1997, cited in Ellis, 2005, p. 19). Others have argued that these approaches are too structured and that they might hinder the creativity of instruction (e.g., Manouchehri, 1998; Simon, 1997; Wakefield, 1997). However, the present investigation will later provide sufficient evidence for adopting the two approaches in teaching practice because students benefit significantly from their use, particularly students with learning difficulties.

2.2.1.2 The Cognitive Perspective

The cognitive perspective focuses on the student’s role as an active contributor to the teaching and learning process. Cognitive theory is drawn from the work of many educational philosophers and psychologists (e.g., Ausubel, Novak, & Hanesian, 1968; Piaget & Inhelder, 1969; Vygotsky, 1978). Adherents of this approach focus on “the study of perceptual processes, problem-solving abilities and reasoning abilities” (Ellis, 2005, p. 20). Teaching approaches based on the cognitive perspective generally encourage students to work cooperatively rather than individually, and emphasise the development of problem-solving skills more than instructional procedures. In this regard, the teacher’s role in the teaching and learning process is as a facilitator rather than as an instructor.

2.2.2 Teaching Approaches for Students with Learning Difficulties

Two teaching approaches involving direct instruction and strategy instruction have been documented as having pervasive influence for remediating students’ learning difficulties by meta-analytic research on educational psychology (Ellis, 2005, p. 28). This section provides an overview of the two teaching approaches, together with their methods of implementation and the reasons why these approaches are appropriate for students with learning difficulties. Brief descriptions of ‘direct instruction’ and ‘strategy instruction’ are provided in Section 2.2.2.1 and 2.2.2.2 as follows.

2.2.2.1 Direct Instruction

Direct instruction, also referred to as explicit instruction, can be defined as:

> … a systematic method for presenting material in small steps, pausing to check for student understanding, and eliciting active and successful participation from all students. (Rosenshine, 1986, p. 60)

Direct instruction, therefore, is drawn from behavioural theory which asserts that learning would be greatly enhanced if the teacher could give clear instructional presentations, eliminate misinterpretations and enable generalisations (Northwest Regional Educational Laborary, 2003). Adherents of the direct instruction approach assume that all students are able to learn. In this regard, they believe that the student's failure in learning essentially comes from a deficiency in teacher instruction (Ellis, 2005, p. 29).

Research conducted over 40 years has provided strong evidence in support of the notion that teaching methods involving direct instruction are some of the most successful teaching practices for improving student academic performance, in particular, for students experiencing difficulties in learning (e.g., Forness, 2001; Swanson & Hoskyn, 1998). In particular, research on the direct instruction approach focusing on the numeracy domain concluded that direct instruction is an effective teaching approach for mathematics teaching, especially in the case of students with learning difficulties (e.g., Kroesbergen & Van Luit, 2003; Kroesbergen, Van Luit, & Maas, 2004; Miller, Butler, & Lee, 1998; Westwood, 2004). Some studies (e.g., Grossen & Ewing, 1994; Kroesbergen et al., 2004), for example, have compared the effectiveness of the two teaching approaches, direct instruction and constructivist instruction, in mathematics education. The results indicated that in teaching mathematics, particularly for students with learning difficulties, the direct instruction approach was remarkably more effective than the constructivist instruction approach. That might be because the direct instruction teaching approach enables highly structured instruction and large amounts of practice that would work better for students with learning difficulties (Block, Everson, & Guskey, 1995). Casey (1994), however, argued that direct instruction puts more attention on the learning environment than on the causes of learning difficulties.

2.2.2.2 Strategy Instruction

Strategy instruction is generally associated with constructivist approaches. In strategy instruction, teaching is designed to develop meta-cognitive strategies and self-regulation strategies, involving describing the strategy, explaining the reason for using the strategy, discussing how the strategy can be applied and exemplifying the context in which the strategy should be used (Van Kraayenoord, 2004a). Strategy instruction and direct instruction are drawn from different views of student learning. In direct instruction, a student is assumed to be a passive learner who is able to master a range of sub-skills then automatically and routinely apply these to practice, whereas in strategy instruction a student is assumed to be an active learner in the meaning-making process, one who can construct meaning by integrating the existing and new knowledge and use strategies flexibly to master comprehension (Dole, Brown, & Trathen, 1996, p. 74).

A growing body of strategy instruction research focusing on the numeracy domain has suggested that strategy instruction is beneficial to students with learning difficulties in improving their mathematics performance (Ellis, 2005, p. 39). Research comparing the effectiveness of strategy instruction and direct instruction (Tournaki, 2003, p. 449) indicated that strategy instruction has significantly positive effects on students with learning difficulties, compared with direct instruction, whereas both strategy instruction and direct instruction have significantly positive effects on general education students. Tournaki's (2003) study, therefore, indicated that the two teaching approaches have different effects on improving student performance depending on the characteristics of individual students (Purdie & Ellis, 2005, p. 29). Furthermore, Swanson and Hoskyn's (1998, p. 277) meta-analysis demonstrated that both strategy instruction and direct instruction approaches have pervasive positive effects on remediating students experiencing difficulties in learning.

2.2.2.3 A Balanced Approach to Teaching Arithmetic for Students with Learning Difficulties

A balanced approach to teaching arithmetic for students experiencing difficulties in learning which draws from aspects of both direct instruction and strategy instruction has gained currency among educational researchers and practitioners (Purdie & Ellis, 2005, p. 32). The idea of combining instructional approaches was based on the argument that one single instructional approach cannot have sole claim to being 'best practice'. Balanced approaches would select the strongest components of each single approach which would have been informed through research and practice in order to accommodate the diversity of students' needs (Ellis, 2005, p. 44). Research findings suggest that the balanced approach would provide students with the best opportunities for success. A meta-analysis (Swanson & Hoskyn, 1998, p. 308), for example, demonstrated that the balanced approach resulted in the highest effect size in the case of students with learning difficulties. Rather than promoting a single instructional approach, an increasing number of educators have advocated for balancing the direct instruction and strategy instruction approaches (Harris & Graham, 1996; Westwood, 1999a, 2000).

The position taken in the present investigation draws on the balanced approach, which involves a harmonious balance between direct instruction and strategy instruction. This is consistent with the instructional approach described in Section 2.1.2.1, that is, an inquiry-based approach to instruction. The approach has much in common with one involving strategy instruction because both are well grounded in a constructivist theory of learning. As well, a rehearsal approach to instruction has much in common with direct instruction because both are well grounded in the behavioural theory of learning.

2.2.3 Section 2.2 Concluding Remarks

In Section 2.2, an historical overview of general theories of teaching and learning mathematics was provided and effective teaching practices that address learning difficulties in mathematics were described. Also outlined, were the two theoretical models of learning in mathematics, referred to as the behavioural perspective and the cognitive perspective. Two teaching approaches that correspond to the two theoretical models, together with their methods of implementation and the reasons why these approaches are appropriate to students with learning difficulties, were discussed. The review showed that each teaching approach has its strengths and weaknesses in addressing the needs of students with learning difficulties. Thus a balanced approach drawing on elements of both teaching approaches is likely to be successful for students with difficulties in learning arithmetic. The position taken in the present investigation is that instruction should involve a harmonious balance between an inquiry-based approach and a rehearsal approach to instruction (as described in Section 2.1.2.1), which accords with a balanced approach.

2.3 Research on One-to-one Instruction in Mathematics

> Although one-to-one tutoring has been regarded as the most effective method of teaching, surprisingly little is understood about tutoring expertise. Much educational research focuses on classroom teaching, whereas the few studies that focus on one-to-one tutoring do not offer a precise information-processing account of this skill. (McArthur, Stasz, & Zmuidzinas, 1990, p. 1)

2.3.1 Historical Review of the Effectiveness of One-to-one Tutoring

One-to-one instruction, regarded as a complement to classroom instruction, is usually considered to be the most effective way to improve student achievement (Elbaum, Vaughn, Hughes, & Moody, 2000, p. 605). Since the 1970s, school-wide tutoring programs have operated in many schools in order to support students experiencing academic difficulties. Depending on the programs, tutoring instruction could be provided by classroom teachers (e.g., Clay, 1979) or by paraprofessionals and volunteers (e.g., Invernizzi & Juel, 1996). Tutoring instruction could occur within a classroom, for example, by employing teaching assistants to provide additional support for such students (Dowker, 2004, p. 21), or outside the regular classroom – these programs being referred to as pull-out programs (e.g., Madden & Slavin, 1987, p. 3).

The effectiveness of one-to-one instruction has been demonstrated by empirical research, particularly for students who are regarded as being unsuccessful in their regular class or who are identified as experiencing difficulties in learning (Bloom, 1984; Jenkins, Mayhall, Peschka, & Jenkins, 1974; Wasik & Slavin, 1993). However, the reasons for the effectiveness of one-to-one tutoring remain relatively unexplored and there is little that is understood about tutoring expertise, particularly with expert tutors (Cade, Copeland, Person, & D'Mello, 2008, p. 470; Lu et al., 2007, p. 456).

2.3.2 Approaches to Studies of the Effectiveness of One-to-one Tutoring

"The effectiveness of one-to-one tutoring in human and computer tutors raises the question of *what* makes tutoring so powerful?" (D'Mello, Olney, & Person, 2010, p. 2). Reasons for the effectiveness of tutoring generally fall into three theoretical categories:

- pedagogical strategies used by tutors;
- students being given opportunities to engage more actively in constructing knowledge; and
- the coordinated effort of both tutor and student.

Accordingly, Chi, Siler, Jeong, Yamauchi and Hausmann (2001, p. 471) formulated three contrasting hypotheses which can account for tutoring effectiveness. These hypotheses are known as the tutor-centred hypothesis (T-hypothesis), the student-centred hypothesis (S-hypothesis) and the interactive coordination hypothesis (I-hypothesis). Rather than promoting a single hypothesis, the present investigation advocates the balancing of the three hypotheses that is at least partially complementary. That is, from the present investigation's point of view, the effectiveness of one-to-one instruction on students' learning necessarily involves: tutors using sound pedagogical strategies; students being given opportunities to engage more actively in constructing knowledge; and cooperation between tutor and student being fostered. The following section describes the three hypotheses in detail and their relationships to each other.

2.3.2.1 Tutor-centred Pedagogy (T-hypothesis)

In tutor-centred pedagogy, tutors basically dominate the dialogue and decide what methods, activities and techniques to use in the tutoring sessions. The T-hypothesis contends that the pedagogical strategies undertaken by the tutors underlie the effectiveness of tutoring (Chi et al., 2001; D'Mello et al., 2010). A typical example of this approach is the 5-Step Tutoring Frame (Graesser et al., 1995). These five steps are described as follows.

1. Tutor poses an initial question;
2. Students responds to the question;
3. Tutor gives feedback based on the student's answer;
4. Tutor scaffolds to improve or elaborate the student's answer; and,
5. Tutor gauges student's understanding.

Research programs of human tutoring based on this T-hypothesis have focused on identifying the repertoire of pedagogical strategies available to tutors that contribute to student learning (e.g., Du Boulay & Luckin, 2001; Fox, 1991; Merrill, Reiser, Ranney, & Trafton, 1992; VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003; Weaver, 2006). These research programs have yielded some significant insights into the pedagogical strategies undertaken by tutors, such as giving feedback, giving explanations and scaffolding. Table 2.2 below summarises the nine relevant studies.

Table 2.2 Summary of study approaches to pedagogical strategies undertaken by tutors

Author(s)	**Academic domain**	**Research aim**
McArthur et al. (1990)	Algebra	Endeavour to determine what, where and when a particular pedagogical strategy is used
Looietal (2005); Merrill, Reiser, Merrill & Landes(1995)	LISP programming	Determine how such strategies generate explanations and feedback
Chi, Siler & Jeong (2004)	Nursing, Biology	How tutors monitor student understanding
Del Solato & Du Boulay (1995); Lepper & Woolverton (2002); Lepper, Woolverton, Mumme & Gurtner (1993); Person et al. (2007)	General	How tutors motivate the students
Fiedler & Tsovaltzi (2003); Hume, Michael, Rovick & Evens (1996)	Mathematics	What variety of hints tutors use

2.3.2.2 Student-centred Knowledge Construction (S-hypothesis)

The S-hypothesis basically contends that tutoring is effective, not necessarily because tutors select a specific pedagogical strategy in a precise and expert way, but because these strategies encourage students to engage actively in constructing knowledge (Chi et al., 2001; D'Mello et al., 2010). Student knowledge construction can be manifested externally by observable behavioural activities such as spontaneously self-explaining (Chi, Bassok, Lewis, Reimann, &

Glaser, 1989), asking questions, drawing, taking notes, summarising, explaining to the tutors and answering the tutors' questions (Chi et al., 2001).

Tutoring effectiveness has been examined largely from the perspective of the tutors (e.g., Evens, Spitkovsky, Boyle, Michael, & Rovick, 1993; McArthur et al., 1990; Merrill et al., 1992), whereas the role of the student in a tutoring context has been given little attention with a few exceptions (Core, Moore, & Zinn, 2003; Fox, 1991; Graesser et al., 1995; P. Jordan & Siler, 2002). Table 2.3 in Section 2.3.3.1 summarises the seven relevant studies.

2.3.2.3 Tutor and Student Co-ordination (I-hypothesis)

The T-hypothesis and S-hypothesis view tutoring effectiveness from either a tutor perspective or a student perspective, although each perspective may acknowledge the role of the other (Chi et al., 2001, p. 480). The T-hypothesis examines tutor moves in response to specific student moves, and the S-hypothesis investigates student constructive responses and their effects on learning when teachers elicited such responses. In contrast, the I-hypothesis states that the effectiveness of tutoring arises from the advantage of tutor-student interactions (Chi et al., 2001, p. 481). Thus interaction is an important feature of the I-hypothesis. This has often been promoted in the context of situativity theory (Durning & Artino, 2011; Greeno & Middle School Mathematics Through Applications Project Group, 1998; Owen, 2004) or in the context of social constructivism (Newman, Griffin, & Cole, 1989; Palincsar, 1998). The advantage of the tutor-student interaction approach is that it apparently enables the student and tutor to co-construct an understanding or a shared meaning that neither partner initially understands (Dillenbourg, 1999; Roschelle, 1992).

2.3.3 Research on One-to-one Tutoring Strategies

Litman and Forbes-Riley (2006, p. 61) examined correlations between dialogue behaviours and student learning in tutoring by analysing dialogue acts between teachers and students. The findings from Litman and Forbes-Riley's study lend some support to each of the three hypotheses above (T-hypothesis, S-hypothesis and I-hypothesis), that is, student learning correlated with student dialogue acts, with teacher dialogue acts, as well as with bigrams comprising both types of acts (Chi et al., 2001, p. 471). The following sections develop this view by outlining strategies that appear to utilise all three hypotheses to some degree.

2.3.3.1 Research on Tutor-student Interactions

Studies on examining tutorial dialogue have identified a range of tutoring strategies that tutors use when interacting with their students. These studies show that tutor-student interactions are so rich and complex that researchers have not yet converged on a shared set of tutoring strategies (Ohlsson et al., 2007, p. 360). This research is reviewed in the following section.

Graesser et al. (1995, pp. 507– 509) identified some common strategies that tutors in their sample used to improve or elaborate students' answers. These strategies include *pumping*, *prompting students to fill in words*, *splicing* and *summarising*. Person (2006) described 23 categories of tutoring strategies including *hint*, *prompt*, *pump*, *bridge*, *summarise*, *ask clarification questions*, *ask comprehension-gauging questions*, *provide counterexamples*, *give direct instruction*, *force a choice*, *provide a preview*, *provide examples*, *complain* and *re-voice*. Although long, this list is not inclusive of tutoring strategies described in other studies. In researching activities responsible for student learning in tutoring, VanLehn et al. (VanLehn et al., 2003, p. 209), for example, used coded tutoring transcripts to record tutors' strategies of *explanations* and *goal-setting hints* in situations in which an impasse had occurred. These strategies differ from any categories in Person's (2006) list as described above.

Another study on expert tutors, Person et al. (2007), provided two sets of tutor moves involving tutor pedagogical dialogue moves and tutor motivational dialogue moves. The set of tutor pedagogical dialogue moves included *direct instruction/explanation*; *simplified problem*; *comprehension-gauging*; *prompt*; *hint*; *splice correct answer*; *new problem*; *pump*; *review*; *summary*; *example*; *paraphrase*; *forced choice*; and *counter-example*. The set of tutor motivational dialogue moves included *positive feedback*; *conversational "Okay"*; *repetition*; *neutral feedback*; *humor*; *negative feedback*; *motivation/solidarity*; and *attribution*.

In describing and illustrating key features of Mathematics Recovery instruction, Wright et al. (2002), provided a set of 12 tutoring strategies which they called 'key elements' of one-to-one teaching. The term key element here refers to a micro- instructional strategy that teachers use during highly interactive one-to-one teaching. The term key element is used similarly to, but not identical with, the term Key Element as used in the present investigation. The set includes *micro-adjusting*, *scaffolding*, *handling an impasse*, *reformulating a task*, *pre-formulating a task*, *introducing a setting*, *post-task wait-time*, *within task setting change*, *screening*, *colour-coding and flashing*; *teacher reflection*; *child checking*; and *affirmation*. A set of seven key elements (Wright, 2010) is a supplement to the first set of 12 key elements of one-to-one teaching described above. The set includes *scaffolding before*; *scaffolding during*; *responding to an incorrect response*; *responding to a correct response*; *querying a correct response*; *explaining*; and *changing a task format*. The set of key elements would seem to provide a sound basis for critical observation and analysis of one-to-one teaching for first-graders (Ewing, 2005; Munter, 2010; Wright, Martland, Stafford, et al., 2006).

Ewing (2005, pp. 29–45) documented the characteristics of one-to-one teaching used by four Mathematics Recovery teachers by analysing videotaped excerpts of their Mathematics Recovery teaching sessions. These characteristics included *scaffolding*, *double bind*, *illusion of competence*, *pre-formulating* and *reformulating questions*, *post question wait-time*, *vague or ambiguous questioning*, *questioning and prompting and communication*. Some of these characteristics are similar to the 'key elements' described above. Ewing (2005, p. 72) found that *scaffolding*, *post question wait-time*, and *questioning and prompting* are important characteristics of one-to-one teaching and that they support the underlying principles of Mathematics Recovery teaching. An experimental model for analysing one-to-one teaching was developed by Ewing (2005) based on those characteristics in a context of intervention with first-graders. However, Ewing did not describe how and why these teaching characteristics are used and why particular characteristics make a difference in student learning.

Aghilieri's (2006) worked on scaffolding practices that enhance mathematics learning, though not specific to one-to-one mediation. Aghilieri described scaffolding practices as three levels involving environmental provisions, explaining, reviewing and restructuring, and developing conceptual thinking.

Munter (2010, p. 48) evaluated the effectiveness of some key elements of one-to-one Mathematics Recovery teaching such as post-task wait-time and child checking in relation to student outcomes. He found that these key elements have a significant effect on student learning. Furthermore, Munter (2010, p. 50) described *re-voicing*, *different strategy* and *compare strategies* as instances of 'positive infidelity' (Cordray & Hulleman, 2009), that is, a teacher uses a non-Mathematics Recovery aspect of tutoring but there is a positive impact on student outcomes. *Re-voicing* refers to a tutor re-voicing, that is, repeating a student's strategy. *Different strategy* refers to a tutor asking the student to solve a problem in a different way. C*ompare strategies* refers to a tutor questioning the student to encourage him or her to examine the mathematical similarities and differences between two or more strategies. Munter found that the mean frequencies of the use of strategies with positive infidelity were very low: 0.03, 0.01 and 0.005 for *re-voicing*, *different strategy* and *compare strategies* respectively. Munter's study (2010) drew on Mathematics Recovery instruction in the United States and focused on Year 1 students. In the present investigation, the strategies with positive infidelity are examined in relation to their occurrence across the participating teachers in the data set which draws on the Mathematics Intervention Specialist Program and focuses on Year 3 and Year 4 students. Table 2.3 summarises of the main results from studies of tutoring strategies.

Table 2.3 Summary of results from studies of tutoring strategies

Author(s)	Academic domain	Expert or non-expert tutors	Main results
Graesser at el. (1995)	Undergraduates in research methods and Year 7 in Algebra	Non-expert	Identified common strategies that the tutors used to improve or elaborate the student's answer, e.g., pumping; prompting students to fill in words; splicing; summarising
Wright at el. (2002)	Maths Year 1	Expert	Described key elements of one-to-one teaching: micro-adjusting; scaffolding; handling an impasse; reformulating a task; pre-formulating a task; introducing a setting; post-task wait-time; within task setting change; screening; colour-coding and flashing; teacher reflection; child checking; affirmation
VanLehn et al. (2003)	Physics	Expert	Student learning occurs when an impasse is reached and that coordinated with some tutor strategies such as explanations and goal-setting hints
Ewing (2005)	Maths Year 1	Expert	Described key elements of one-to-one teaching: scaffolding; double bind; illusion of competence; pre-formulating and reformulating questions; post question wait-time; vague or ambiguous questioning; questioning and prompting; communication
Person (2006)	Maths and Science Year 7-12	Expert	Identified 23 categories of tutoring strategies, e.g., hint; prompt; pump; bridge; summarise
Person et al. (2007)	Math and Science Year 7-12	Expert	Tutor motivational dialogue moves includes positive feedback; conversational "Okay"; repetition; neutral feedback; humour; negative feedback; motivation/solidarity; and attribution.
Wright (2010)	Maths Year 1	Expert	Described seven key elements: scaffolding before; scaffolding during; responding to an incorrect response; responding to a correct response; querying a correct response; explaining; changing a task format

In Table 2.3, it is evident that tutoring research has not converged on a widely agreed-upon theory of how the tutoring strategies should be segmented and characterised (Ohlsson et al., 2007, p. 360). There is a need to study systematically tutoring strategies in a particular academic discipline.

2.3.3.2 Sophisticated pedagogical strategies in one-to-one tutoring

Cade et al. (2008, p. 476) listed theoretical models and strategies preferred by expert tutors. These are presented in Table 2.4 as follows.

Table 2.4 Sophisticated tutoring strategies

Sophisticated tutoring strategy	Reference
Modelling-scaffolding-fading	Rogoff & Lave (1984)
Socratic tutoring	Collins (1985)
Contingent teaching (A ZPD approach)	Du Boulay & Luckin (2001)
Inquiry teaching	Dillon (2004)
Situated learning	Collins, Brown & Holum (1991)
Sophisticated motivational techniques	Lepper, Woolverton, Mumme & Gurtner (1993); Person et al. (2007)
Error diagnosis and correction	VanLehn (1990)
Anchored situated learning	Bransford, Goldman & Vye (1991)

Studies focusing on non-expert tutors found that the sophisticated pedagogical tutoring strategies referred to above are non-existent in their research corpus (Graesser et al., 1995, p. 502; Graesser, Wiemer-Hastings, Wiemer-Hastings, Kreuz, & Tutoring Research Group, 1999, p. 39; Person & Graesser, 2003, p. 337). These researchers suggested that tutors need to be trained to be able to use the sophisticated tutoring strategies. The present investigation drew on expert tutors. Accordingly, a discussion of the use, if any, of these sophisticated tutoring strategies, by the participants when interacting with a student in the course of teaching was addressed in Chapter 7.

2.3.3.3 How the present investigation fits with relevant prior research

A review of research on examining tutorial dialogue in Section 2.3.3.1 showed that the terms used to refer to tutoring strategies that expert tutors use when interacting with students vary from study to study. These terms include characteristics of one-to-one teaching (Ewing, 2005);

key elements (Wright, 2010; Wright et al., 2002); tutor moves (Lu et al., 2007); and dialogue moves (Person et al., 2007).

Decades of research on examining one-to-one instruction have shown that tutor-student interactions are complex and researchers have not yet converged on a common set of expert tutoring strategies (e.g., Graesser at el., 1995; Lu at el., 2007; McMahon, 1998; Person at el., 2007; Wright, 2010; Wright at el., 2002). As well, tutoring research has not converged on a widely agreed-upon theory of how tutoring strategies should be segmented and characterised (Ohlsson et al., 2007). Further, the research reviewed earlier focused on different academic disciplines. In the particular case of one-to-one instruction in whole-number arithmetic, there is still a limited amount of research literature (Kyriacou & Issitt, 2008). Whereas earlier research on teacher behaviours in Mathematic Recovery (Wight, et al., 2002; Ewing, 2005; Wright, 2010) focused on early arithmetic, mainly involving Year 1 students, the present investigation focuses on identifying and illuminating the nature of Key Elements of one-to-one intervention instruction related to whole-number arithmetic with Year 3 and Year 4 students.

Previous studies mainly took a tutor-centred approach, which tacitly assumed that tutoring effectiveness arises from the tutors' strategies (Chi et al., 2001), except for a few recent studies that have emphasised the role of teacher-student interactions in developing students' conceptual understanding and knowledge construction (e.g., Grandi & Rowland, 2013; Leatham, Peterson, Stockero, & Van Zoest, 2015; Lester, 2007). The Key Elements established in the present investigation build on students' mathematical thinking to develop mathematical concepts which are valued by the mathematics education community (Leatham et al., 2015).

Cade et al. (2008, p. 470) claim that research on examining tutoring strategies in expert tutoring failed to provide the context necessary for understanding how a cluster of tutoring strategies relate to each other. In the present investigation, after identifying and illuminating the Key Elements, a framework of Key Elements for analysing one-to-one instruction is conceptualised. The framework will provide the context necessary for understanding how teachers use specific clusters of Key Elements to achieve particular pedagogical goals. The present investigation aims to identify and describe the Key Elements through the following two research questions.

Research Question 1: What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?

Research Question 2: How can Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

2.3.3.4 Expert versus non-expert tutors

Research on expert versus non-expert tutors has documented that most of the tutors in school systems were peer-tutoring, paraprofessional tutors or adults volunteers rather than expert tutors (e.g., Chi et al., 2001; Cohen et al., 1982; Graesser, D'Mello, & Cade, 2010). As Person (2006) documented, only a few expert tutors have been studied (Di Eugenio et al., 2006; Lepper & Woolverton, 2002; Lepper et al., 1993). Regarding research on expert tutor versus non-expert tutor, Person et al. (2007, p. 2) stated that:

> Decades of research on human tutoring have elucidated our understanding of the tutoring process. The majority of these studies, however, have primarily focused on untrained or "typical" tutors and have provided little insight into the strategies used by expert human tutors.

Research on one-to-one tutoring has found that expert tutors are more effective in student learning than non-expert tutors (e.g., Bloom, 1984; Chae et al., 2005; Di Eugenio et al., 2006; Lu et al., 2007). Researchers found that expert tutors yield the highest gains with the effect size of 2 sigma (Bloom, 1984) compared with non-expert tutors (0.4) (Cohen et al., 1982) and Intelligent Tutoring Systems (1.0 sigma) (Corbett, Anderson, Graesser, Koedinger, & VanLehn, 1999).

Ohlsson et al. (2007, p. 351) claim that studies of expert tutors contain some weaknesses related to the number of expert tutors that have been studied and to how the expert tutors are identified. Expert tutors are often identified on the basis of indirect indicators such as the tutors' qualifications and the period of time they have been tutoring (Ohlsson et al., 2007, p. 351). In the present investigation, such weaknesses were overcome by: (a) verifying that the participating expert tutors provided highly effective teaching in terms of student outcomes based on pre- and post-tests; and (b) being chosen by the project leader who was responsible for the specialist training of those tutors.

2.3.4 Section 2.3 Concluding Remarks

In section 2.3.1, a historical review of the effectiveness of one-to-one tutoring showed that one-to-one tutoring is generally regarded as the most effective approach to improving student achievement, particularly for students with learning difficulties. In section 2.3.2, possible reasons for the effectiveness of tutoring were discussed in relation to three hypotheses (T-hypothesis, S-hypothesis and I-hypothesis). The position taken in the present investigation is a balanced approach that involves regarding the three hypotheses to be at least partially complementary. Further, effective one-to-one instruction is assumed to necessarily involve tutors using pedagogical strategies which involve providing students with opportunities to engage actively in constructing knowledge and fostering a cooperative approach between tutor and student.

In section 2.3.3, key findings of research on one-to-one instruction, particularly on tutoring strategies that tutors use when interacting with students, were reviewed. This review was intended to sketch out relationships among the key findings and to identify any gaps in the research.

2.4 Concluding Remarks

This chapter described the conceptual framework that informed the investigation and placed the present investigation in the broader educational and mathematical literature involving perspectives on effective teaching practices that address learning difficulties in mathematics. As well, the chapter described the key findings of research on one-to-one instruction and identified relationships among the key findings and any gaps in the research.

Chapter 3 – **Methodology**

The present investigation used a particular research methodology for observing and recording the incidence and consequences of the adoption of certain instructional strategies in one-to-one intervention teaching with Year 3 and Year 4 students about whole-number arithmetic. This research methodology, which requires the close examination of existing video recordings, is relatively straightforward. The video files were created as part of a Mathematics Intervention Specialist Program (Wright et al., 2011), which itself forms part of a larger database that has been assembled over the past decade in Australia and elsewhere for use in professional development programs for primary school teachers.

This chapter explains the research methodology and it documents how data for the investigation were collected. The research methodology is broadly phenomenological (Van Manen, 1990, 1997) in approach. Some aspects of grounded theory (Glaser & Strauss, 1967; Strauss & Corbin, 1994, 1998) were also employed.

The first part of the chapter introduces the research methodology for the investigation and explains why it was selected. The particular method for selecting and observing cases is then explained, and the data set is described in the context of the broader Mathematics Intervention Specialist Program. The process of implementing the investigation and of analysing the data is reported. Finally, relevant trustworthiness criteria are considered.

3.1 Selection of a Methodology

The researcher has long been concerned about students with learning difficulties in whole-number arithmetic, particularly in Years 3 and 4, and with developing an understanding of the kinds of approaches available to address their difficulties. An impressive and enlightening approach is the one-to-one intervention teaching approach associated with the Mathematics Intervention Specialist Program. As explained in Chapter 1, in the Mathematics Intervention Specialist Program, every teaching session and assessment interview was video-recorded for subsequent analysis in teachers' workshops associated with the professional development program integral to the Mathematics Intervention Specialist Program.

The present investigation seeks first to identify and illuminate Key Elements of one-to-one instruction with Years 3 and 4 students, and then to conceptualise a framework for the analysis of one-to-one instruction in the domain of whole-number arithmetic. In the present investigation, video recordings of teaching sessions and assessment interviews that were conducted as part of the Mathematics Intervention Specialist Program provide the data required for the analysis. A challenge for the investigation is to transform the 'lived experience' of the participating teachers and the students in the context of one-to-one intervention instruction into a 'textual expression' in the form of Key Elements of one-to-one instruction. A process of repeat analysis of selected cases from the data bank is employed for the purposes of illuminating the nature of the Key Elements.

The present investigation is concerned with examining teacher behaviours in a deep and prolonged way. To have analytical bite, an approach to developing thick rich descriptions of behaviours is needed. Further, since the investigation is essentially constructivist in its approach, criteria for rigour must be operationalised in the data collection and analysis processes. The methodology of phenomenology, as described by Van Manen (1997), is employed, drawing also upon the criteria outlined by Lincoln and Guba for rigour in constructivist research. While Morse et al. (2002) are critical of the terminology of trustworthiness criteria, as explained by Lincoln and Guba (1985), and question whether the trustworthiness criteria might apply to phenomenology, it was considered that these constructs are essential to ensuring the credibility, dependability, transferability and confirmability of the findings from the present investigation. The operationalization of these criteria is explained later in this chapter.

A deeper understanding of the nature of particular Key Elements was sought by observing and describing the filmed behaviours of different teachers in different one-to-one teaching settings until the nature of the teaching behaviours could be distilled and described. In this way, it was possible to explore the participating teachers' and students' lived experience of Key Elements. Central to understanding how Years 3 and 4 students learn mathematical concepts is the notion that mostly they cannot explain fully in words what they are experiencing during the learning process, in this case, mastering whole-number arithmetic. Also, when teachers are asked about their experience of using specific Key Elements, they find it difficult *post hoc* to describe in full detail, moment to moment, what they have done in their teaching. As Polanyi (1966, pp. 5-6) argued, we know much more than we can express in words, and we usually cannot describe in detail how we know it. It is, therefore, not practicable for the researcher to identify and document the essence of the phenomena simply by interviewing teachers or students. Ontologically, there is a need to view the teaching practices and the interactions closely, intensively and repeatedly in order for the researcher to be able to notice and interpret significant moments and events in the context of interaction between the teacher and the student. Such close examination over time may be expected to provide access to the conveyance of knowledge as it is experienced within teacher-student dyads. It requires the researcher to step back and reflect on the behaviours of the teachers, beginning with existing well-documented Key Elements (Wright at el., 2002; Wright, 2010) and the behaviours they elicit from their students. It was decided to use existing video recordings of teacher-student dyads from the target group so that the phenomena could be reviewed often and deeply considered to identify which teacher behaviours trigger learning gains. A phenomenological (see, for example, Van Manen, 1990) approach to understanding tacit Key Elements and how they might trigger tacit learning gains was, therefore, selected as the methodology for the investigation.

Phenomenology, as described by Van Manen (1990), serves perfectly to meet the need to describe the essence, or tacit communication and tacit knowing, triggered by Key Elements. The neatness of fit in phenomenology as a methodology in the present research derives from its capacity to permit repeated observation and examination of certain teacher and student behaviors that lead to learning gains in mastering whole-number arithmetic among a target sample of Years 3 and 4 students. In addition, the basic phenomenological technique is to reduce individual experiences of the participating teachers to their behaviors that constitute Key Elements, which in turn lead to the development of thick, rich descriptions of what Van Manen (1990, p. xiv) termed the 'universal essence' of the Key Elements. The approach sought to develop a comprehensive description of the essence of Key Elements: What are the meanings and significance of the Key Elements observed? How did teachers behave when implementing the Key Elements in the context of one-to-one intervention teaching? What responses did teacher behaviours trigger in their students?

3.2 Phenomenology as a Methodology

There is a strong philosophical component to phenomenology, as Creswell (2007) explains. It draws heavily on the works of Edmund Husserl (1859–1938), a German philosopher and mathematician. Other philosophers who expanded on Husserl's view are Heidegger, Gadamer, Sartre, Alfred Schutz, and Merleau-Ponty (Spiegelberg, 1982). Alfred Schutz (1899–1959) was an important influence in applying and establishing phenomenology as a major social science perspective (Schutz, 1977). The literature points to different philosophical arguments for the use of phenomenology. One approach follows in the footsteps of Husserl (Husserl, 1913/1982), whose approach is often referred to as 'pure phenomenology'. Heidegger (1962) espoused a different school of thought. Husserl focused on 'understanding beings or phenomena', whereas Heidegger focused on 'the mode of being human' or 'the situated meaning of a human in the world', as Laverty (2003, p. 24) described it. Husserl emphasized the acts of attending, perceiving, recalling, and thinking about the world. For Husserl, human beings were understood primarily as knowers as Laverty (2003) explained. In contrast, Heidegger viewed humans primarily as being creatures concerned with an emphasis on their fate in an alien world, according to Annells (1996).

Considering these perspectives, Creswell (2007, p. 58) indicated that "the philosophical assumptions rest on some common ground: the study of the lived experiences are conscious experiences as Van Manen (1990) explained, involving the development of descriptions of the essences of these experiences, but not explanations or analyses (Moustakas, 1994)". Two approaches to phenomenology which correspond with the two philosophical assumptions described above are highlighted in literature (see, for example, Creswell, 2007). These are hermeneutic phenomenology as described by Van Manen (1990) and transcendental phenomenology as described by Moustakas (1994). Different approaches allow different nuances of focus. Transcendental phenomenology focuses on the essential meanings of individual experience, whereas hermeneutic phenomenology (Van Manen, 1990) focuses on the language and structure of communication.

Although there are various forms of phenomenological approaches, they have in common an emphasis on exploring "how human beings make sense of experience and transform experience into consciousness, both individually and as shared meaning" (Patton, 2002, p. 104). The current investigation adopts Patton's (2002) broader meaning of phenomenology, and essentially draws on the phenomenological approach described by Van Manen (1990, 1997) because it accommodates both perspectives. It is neither practicable nor relevant to the present investigation to differentiate between the essential meanings of experience and the language and structure of communication. Both have equal relevance in providing richly layered descriptions of teaching behaviours that are both verbal and conveyed by tacit means. As Polanyi (1966, p. 4) observed, we should start from the understanding that "we can know more than we can tell". It is implied that some facts seems obvious enough, but many conveyances of meaning are difficult to express verbally, nor would one wish to do so, as Becher (1989) noted in relation to academic disciplines. A nudge, a wink or a stern look convey much more than their verbal representations. Van Manen's (1997) approach is to constantly reflect on the phenomenon's essence, until that which is tacit, can be described and undersood. Van Manen (1997, p. 10) explained that the nature of a phenomenon is universal and this can be described through an investigation of the meaning and structure that governs the cases and particular manifestations of the nature of the phenomenon. He suggested that "the essence or nature of an experience has been adequately described in language if the description reawakens or shows us the lived quality and significance of the experience in a fuller and deeper manner" (Van Manen, 1997, p. 10). In this way, phenomenology may be viewed as a systematic endeavor to discover and portray the structures, and the insights, of lived experience (Van Manen, 1997) of a phenomenon. Phenomenologists emphasise, therefore, the description of the commonalities that all participating individuals share when they experience a phenomenon. Regarding the purpose of phenomenology, Creswell (2007) stated that phenomenology reduces individual experiences of a phenomenon to a description of the universal essence, that is, a "grasp of the very nature of the thing" (Van Manen, 1990, p. 177). Van Manen suggested that in this way the universal essence of a phenomenon may only be intuited or grasped through an investigation of the individuals who are encountered in lived experience.

The point of phenomenological research is "to 'borrow' other people's experiences and their reflections on their experiences in order to better be able to come to an understanding of the deeper meaning or significance of an aspect of human experience, in the context of the whole of human experience" (Van Manen, 1997, p. 62). Van Manen argues that we need to gather other people's experiences because those experiences allow us to become more experienced ourselves, in so doing, allowing us to become informed, shaped or enriched by the experience. This process enables people to be able to render the full significance of the structure and meaning of the experience.

The most basic philosophical assumption of Husserl (1913/1982) in phenomenology is that humans can only know what they experience by attending to insights and meanings that awaken their conscious awareness (Patton, 2002, pp. 105–106). Cognitive processes initially start from sensory experience of phenomena, but that experience then needs to be described, explicated and interpreted. Patton (2002, p. 106) argues that there is an intertwinement between descriptions of experience and interpretations. He explains that interpretation is crucial to understanding experience and the experience involves the interpretation. The incorporation of the subjective experience and the objective thing becomes a person's reality. Phenomenologists, thus, focus on making meaning of the nature of human lived experience. In this way, the phenomenological approach is used to study the lived or existential experience and to endeavor to describe and interpret these experiences to a certain level of depth and richness.

A good phenomenological description, as understood by Van Manen (1997), is a sufficient elucidation of some features of the lived world that resonates with our sense of lived life. In other words, "a good phenomenological description is collected by lived experience and recollects lived experience – is validated by lived experience and it validates lived experience" (Van Manen, 1997, p. 27). Therefore, a good phenomenological description comprises the nature of a lived experience that is interpreted. The structure of this experience thus is documented in such a fashion that those reading about it are able to grasp the nature and significance of the experience, or to identify with the description. In this investigation, the term "description" is used to refer to both the interpretive and the descriptive elements of phenomenological analysis.

The following questions are central to understanding the essence of Key Elements.

1. What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?
2. How can Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

The information gained will illuminate the essence of Key Elements, as described and interpreted by using the phenomenological approach outlined above.

According to Patton (2002) there are thirteen tenets of qualitative enquiry which must be reflected in data collection and analysis. In the present investigation all of these were met. First, qualitative studies must take place in their natural settings because context influences meaning. Humans are the research instrument of analysis because qualitative studies deal with the nature of human experience. Therefore, the utilization of tacit knowledge is inescapable. Qualitative methods of data collection and analysis are more appropriate than positivist, measurement-related methods; the human is the instrument of analysis. Data analysis is inductive rather than deductive. Studies seek to illuminate the nature of a phenomenon, so sampling is purposive in order to explore the full scope of issues concerned. Theory emerges (is grounded) in the data, so the research approach is not preordained or contrived to fit particular categories. The research design emerges over time, so the researcher needs to be flexible and open to changing direction if the data signals an unexpected direction. The flow of research stages is, therefore, non-linear, and non-sequential. Further, data collection and analysis occur in tandem because, as findings emerge, new directions in enquiry open up. The natural mode of reporting findings tends to be by case study, which provides thick, information-rich cases with which those reading a study's report may be able to identify. Idiographic interpretation replaces nomothetic interpretation. While nomothetic laws are those governing positivist deductive logic, as in science, idiographic interpretation concerns interpreting the nature of human experience, which cannot be measured.

Applications are tentative and pragmatic, so that if a reader identifies with what is found, then they may tentatively apply it to a different setting. Finally, although Morse et al. (2002) disagree about their application to phenomenological investigations, Lincoln and Guba's (1985) trustworthiness criteria provide a sensible and appropriate replacement for conventional, positivist understandings about reliability and validity.

3.3 Data Source and Data Set

In the present investigation, the focus is on the lived experiences of teachers during one-to-one interactions with Years 3 and 4 students.

3.3.1 The Nature of Observation in the Investigation

"Close observation", as described by Van Manen (1997, p. 68), is used to observe video recordings of individual teachers and students in the context of one-to-one intervention teaching of whole-number arithmetic. The aim is to capture the essence of the Key Elements used and to document the evidence of the progress that the students make during the intervention sessions. In this investigation, observation that is both intensive and extensive is used to study teacher-student interactions and to develop rich descriptions of the Key Elements from the observed data. Several roles of the observer in such cases are identified in the literature. These include: complete participant, participant-as-observer, observer-as-participant, and complete observer, according to Cohen et al. (2011, p. 457). The role of the researcher in this investigation is that of a completely detached observer because the researcher analyses video recordings of teaching sessions that are the documentary files from an already conducted Mathematics Intervention Specialist Program. This role is closest to the traditional ideal of the 'objective' observer identified by Adler and Adler (1994).

This investigation was conducted with a full Human Research Ethics Committee approval from Southern Cross University. The operationalization of the investigation is outlined there in practical terms (see Appendix 2) for the full application and the approval letter. What follows here are the conceptual and practical steps taken in the conduct of the investigation.

"Naturalistic observation", as described by Adler and Adler (1994) and Punch (2005, p. 185), fits neatly with Lincoln and Guba's (1985) explanation of the importance of the natural setting in that the behaviours of the participating teachers and students are in no way stimulated or influenced by the researcher. Further, it is noteworthy that the teaching and learning situations being observed in the investigation were recorded for the professional development purposes in the Mathematics Intervention Specialist Program, not for research purposes. Therefore, the observational strategy in this investigation may be seen as wholly 'unstructured'. Such observations are thus able to be made in a more natural, open-ended way, instead of using predetermined categories and classifications. Whatever the behaviors in the teacher-student interactions might be, these are able to be observed as “the stream of actions and events as they naturally unfold,” which Punch (2005, p. 185) argues is important because the researcher does not influence the data source. Indeed, no pre-existing categories or classifications for describing and analysing video recordings are brought to the investigation or imposed on the observational data source at the start. Instead, these, were expected to emerge as the investigation progresses.

It was also expected that, as the investigation progressed, the nature of observation might change to accommodate a sharpening in the focus of the investigation, in order to lead to ever-clearer research questions. As interesting or important themes emerged, more particular observations are required. This observational data analysis continued until theoretical saturation was reached, according to both Lincoln and Guba (1985) and Adler & Adler (1994). Data saturation was reached, when the video recordings of teaching sessions produce no new perspective on the phenomenon under scrutiny. By using an unstructured approach to the analysis of the data source, the focus was holistically and macroscopically, on the patterns of interaction between the teachers and their students, and therefore upon the behavioral expressions of the teachers and the students.

3.3.2 The Rationale for Using Video as a Data Resource

Observing video files which have been created in a natural setting is crucial for learning the instructional techniques used in Mathematics Recovery as developed by Wright (1994a). In this regard, Hall (2000) suggested that viewing video recordings of teachers in action provides a great opportunity to learn about the detailed structure of teaching and learning. Consistent with the two statements above, Clement (2000) stated that using video as a data resource allows a researcher to capture rich behaviours and complex interactions and also to re-examine data repeatedly. Moreover, Phelps, Fisher and Ellis (2007, p. 186) outline a range of benefits of using video files for data collection. They assert that video files allow: a researcher to make observations without participating, therefore supporting more naturalistic fieldwork; provide insights into interactions such as gesture, eye movement, manipulation of materials or use of computers; and enable a researcher to watch an action sequence repeatedly, thus allowing micro-analysis of interactions.

In the mathematics education research community, the use of video recordings of one-to-one interactions between teacher and student are well accepted as a powerful and extensive tool due to the capacity to record the moment-by-moment unfolding of sounds and sights of a phenomenon as several researchers attest (e.g., Mousley, 1998; Powell et al., 2003, p. 406). Powell et al. (2003), for example, explain that by using video records as data, researchers are able to provide fascinating descriptions of interactions between teachers and students in both clinical and classroom settings involved in an array of mathematics tasks. They go on to indicate that video recordings provide a range of advantages for data analysis due to their permanence as data source artifacts, and because video files can be used repeatedly, offering the potential to enhance triangulation in data analysis. For the reasons outlined above, observing and analysing video records are particularly suited to the proposed investigation. There are many examples of using the process of observation described above in the particular case of students learning of whole-number arithmetic (Steffe & Thompson, 2000a; Wright, Martland, Stafford, et al., 2006).

3.4 Data Collection Procedures

3.4.1 The Data Set

As explained earlier, the primary data for the present investigation drew on the video recording database of the Mathematics Intervention Specialist Program. The intervention program in the Mathematics Intervention Specialist Program had four stages including: school assessments, individual pre-assessments, a teaching cycle, and individual post-assessments (Wright et al., 2011). A description of each phase now follows.

In each school participating in the Mathematics Intervention Specialist Program, the whole cohort of students in the year level underwent several assessments in order to select 12 low-attaining students for participation in the intervention program. The Westwood one-minute tests of basic facts (Westwood, 2000, p. 108) were administered to Years 3 and 4 students, involving multiplication and division tests, as well as addition and subtraction tests. Other assessments commonly used included Success in Numeracy Education (SINE) assessment instrument (CECV, 2002) and the Progressive Achievement Test (ACER, 2005). Those screening assessments have the purpose of identifying students experiencing significant difficulties in learning mathematics, particular in number learning. These tests are designed to be quick and easy to administer and interpret, but do not necessarily provide detailed information about the extent of the student's current mathematical knowledge or the nature of any difficulties they are experiencing.

A one-to-one Videotaped Interview-based Assessment (VIBA) instrument, as devised by Ellemor-Collins & Wright (2008), Munter (2014), Wright (2008, p. 217), and Wright, Martland and Stafford (2006), was administered to the 12 selected students from each participating school. VIBA is a distinctive assessment approach compared with other assessment approaches where the teacher does not videotape the interview but instead writes notes as the interview proceeds. With VIBA, the teacher can give all of her/his attention to posing tasks, observing, reflecting, making appropriate comments, and asking follow up questions. The purpose of VIBA is to learn as much as possible about the student's current mathematical knowledge and strategies, or in other words, to find the cutting-edge of the student's mathematical knowledge. Typically this will involve finding out both what the student can and cannot do. A description of the VIBA is available at Appendix 3.

In their pre-assessment, Years 3 and 4 students were given assessments focusing on the following topics: (a) number words and numerals; (b) structuring numbers 1 to 20; (c) conceptual place value; and (d) addition and subtraction to 100. In the Mathematics Intervention Specialist Program assessments, tasks on the schedules could be used in a flexible way, in response to the student's responses.

Eight of the 12 students participated in teaching cycles of 10 to 12 weeks' duration, involving one-to-one instruction. The teaching cycles were intensive, highly interactive and involved three or four teaching sessions per week, each typically of 30 minutes' duration. A post-assessment was given to all 12 students at the end of the teaching cycle. The learning domains assessed in the pre-assessment tasks were again assessed in the post-assessment tasks. As well, the additional domain of addition and subtraction to 100 was assessed. All of the teaching sessions and pre- and post-assessments were videotaped for subsequent analysis. The data used in the present investigation were selected from the video recordings of four teachers and six students who participated in the Mathematics Intervention Specialist Program.

3.4.2 Participants

The primary data set for this investigation were drawn from the Mathematics Intervention Specialist Program (Wright et al., 2011) in which teachers provide intensive, one-to-one instruction to six low-attaining Years 3 and 4 students. At the beginning, nine teacher-student dyads involving six teachers and nine students was supposed to use for the investigation. However, when six teacher-student dyads was analyzed, it was revealed that further analysis of the data was not generating new findings related to the Key Elements. A decision was made for using six teacher-student dyads involving four teachers and six students.

The basis for selecting a teacher-student dyad was that the relevant teacher was considered to be an effective teacher, in that the relevant student was considered to have made very good progress in learning whole-number arithmetic. Two teachers each taught two students singly and the other two each taught one. The four teachers were selected from a pool of approximately 50 teachers in the Mathematics Intervention Specialist Program and were regarded by the Mathematics Intervention Specialist Program leaders as being particularly competent in intervention teaching. Thus the data involves six sets of video recordings of teaching sessions. Each set consists of up to nine teaching sessions, each of 25-45 minutes' duration, with supplementary materials in the form of pre- and post-assessment interviews for each teacher-student dyad. This resulted in approximately 33 hours of video for analysis.

Theoretically, the basic for selecting the research participants was based on 'purposeful sampling' strategies described by Lincoln and Guba (1985). According to the researcher's request the teacher-student dyads were recommended by the Director of the Mathematics Intervention Specialist Program. These participating teachers were considered to be experienced teachers in terms of the period time that they had working in the program as specialist teachers; and to be strong teachers in terms of their students' very good progress in learning whole-number arithmetic. As well, an 'intensity sampling' was used to select 'information-rich' cases, which actively encouraged (see, for example, Patton, 2002, p. 242) in relation to discovering a 'good' description of the Key Elements.

In the present investigation, the participating teachers were regarded as expert tutors. The indicators that were taken to signify 'expertness' were: (i) having undertaken the professional development program that associated with the Mathematics Intervention Specialist Program in order to be specialists in one-to-one instruction; and, (ii) being one of the four teachers chosen by the Mathematics Intervention Specialist Program leaders because of their students showing significant improvement.

For each teacher-student dyad, teaching sessions were selected from those conducted in teaching cycles of 12 weeks' duration. All the teaching sessions in the teaching cycles were reviewed in order to select seven to nine teaching sessions for each teacher-student dyad. The basis of selection was some teaching sessions from the beginning, some teaching sessions from the middle and some teaching sessions from the end of the teaching cycle. Also, the selection endeavored to cover each of the relevant learning domains, with similar amounts of time for each of the teacher-student dyads.

Prior to commencing the professional development program, all teachers participating in the Mathematics Intervention Specialist Program gave approval for research to be conducted using their video data. This permission extended to use of the data by PhD candidates. As well, the research conducted for this investigation was approved by the Ethics Committee of Southern Cross University.

3.5 Processes of Observation and Data Analysis

The central 'phenomena' of interest to this investigation concern the Key Elements of one-to-one instruction, focusing on whole-number arithmetic with Years 3 and 4 students. Bentz and Shapiro (1998, p. 104) suggested that "doing phenomenology" means capturing "rich description of phenomena and their settings", that is, the researcher must allow the data to emerge.

The first research question was: what Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program? The focus of the second research question was: how can the Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

While in the first research question, the Key Elements were identified individually, in the second research question, the identified Key Elements were seen in an instructional context – a task block. This provided a context necessary for understanding how a teacher used a specific cluster of Key Elements to achieve particular pedagogical goals. As explained in Chapter 1, a task block is a part of a segment which starts at the point where a teacher poses a task and ends when the student solves the task. The focus of the second research question was: how can the Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

A standard method for analysing the data in the present investigation was "close observation" (Van Manen, 1997, p. 68), in which, Key Elements were viewed as the central phenomenon requiring exploration and understanding. The analytical techniques described by Van Manen (1990, 1997) and further elaborated as procedures for phenomenological analysis by Hycner (1999) were adopted. As well, the methodological approach described by Cobb and Whitenack (1996) and Powell et al. (2003) for analysing large sets of video recordings were adopted for data analysis. Additionally, the interpretation and assessment of student progress is based on the Learning Framework in Number and models of learning (Wright et al., 2006) (see Appendix 4), which drew on a range of research (e.g., Cobb & Wheatley, 1988; Cobb, Wood, & Yackel, 1991; Steffe & Cobb, 1988; Wright, 1994b; Wright et al., 2012).

Key Elements of one-to-one instruction were used as an "abiding concern" (Van Manen, 1997, p. 31), being at the heart of the present investigation. In the process of data analysis, essential themes that comprised the essence of Key Elements were reflected upon and documented. The written descriptions of the Key Elements were developed and a strong relation between the Key Elements and students' learning was maintained.

For each teacher-student dyad, the video recordings of teaching sessions were observed closely and repeatedly to characterise each teaching moment in terms of the teacher's instructional strategies in chronological order. The analytical framework for investigating the Key Elements used a sequence of thirteen interacting, non-linear phases.

1. viewing attentively the video recordings of the teaching sessions;
2. transcribing the video recordings;
3. summarising the teaching sessions;
4. bracketing and phenomenological reduction;
5. observing the video recordings of the teaching sessions for a sense of the whole;
6. delineating units of general meaning;
7. delineating units of meaning relevant to the research question;
8. determining themes from the units of relevant meaning;
9. clustering of units of relevant meaning to form 'parent-themes';
10. determining themes from clusters of relevant meaning;
11. identifying general and unique themes for all the teacher-student dyads;
12. contextualisating themes;
13. writing composite descriptions of the Key Elements identified.

Each of these phases of the analytical framework is described in detail in the following chapter.

3.6 Trustworthiness

Van Manen (1997) argued that it should be accepted that human science carries its own criteria for precision, exactness and rigour. From a positivist perspective, precision and exactness are often seen to be indications of a shared single reality which is objective, quantifiable, and generalizable to the population at large, and demonstrable and repeatable. In contrast, human experiences, feelings and values are not measurable. Therefore, Lincoln and Guba (1985) designed trustworthiness criteria for their espoused methodology, Naturalistic Inquiry, in order to ensure 'precision' and 'exactness.' Within phenomenology, a researcher seeks to describe the lived experience of participants as faithfully as possible to document the essence of what happened as experienced by those involved (Giorgi, 1997).

In terms of rigour, Williams and Morrow (2009) suggested three key broad dimensions of trustworthiness for qualitative research. These dimensions concern the integrity of the data, the balance between participant meaning and researcher interpretation, and the clear communication of findings. The integrity of the data refers to acceptable description of research methods and analytic strategies, adequate quality and quantity of data collection, and the sound interpretations of the data. The balance between participant meaning and researcher interpretation requires the researcher to control the bias resulting from their inevitable subjectivity by adopting a position which, while any bias is acknowledged, is set aside in the process of analysis to achieve a construct of *Verstehen* (see, for example, Patton, 2002, p. 52-53). *Verstehen* is a German term that means to understand, perceive, know, and comprehend the nature and significance of a phenomenon. A more detailed approach concerns the trustworthiness criteria developed by Lincoln and Guba (1985) for achieving credibility, transferability, dependability and confirmability.

Credibility

In order to achieve credibility, there was prolonged engagement with the data source, the video files, over an extended period of time. The phenomenological analysis of the data was continuously carried out between (2^{rd} September 2012) and (12^{th} October 2013). The analysis by the researcher involved persistent observation of teacher and student interactions to identify Key Elements and to establish how often they were implemented in teaching sessions. Persistent observation enabled tentative observations of the 'essence' of the Key Elements to be triangulated across teaching settings, across participants, and over time. Yet another trustworthiness method was to undertake regular peer debriefing with the senior researchers and experts in the Mathematics Intervention Specialist Program, to ensure that any observations being made were reasonable, credible and also observable by the supervisors. The academic supervisor for this investigation, Wright (2003, 2008), who originally developed the Mathematics Recovery program, and who has worked and trained hundreds of expert tutors over two decades including the expert tutors in Mathematics Intervention Specialist Program program, acted as key auditor, reviewing formatively the process of data analysis.

Lincoln and Guba (1985) explain the importance of negative case analysis. In the present investigation, whenever there were negative outliers to the observations being made, or negative cases, these were given further scrutiny until their essence became clear. Negative outliers or cases are cues to the idiographic researcher that some data is inconsistent with the rest, so they deserve further scrutiny because this might turn up something new and different or unexpected. The only criterion for credibility that was not conducted was member checking. Member-checking is an essential data collection and analysis tool in post-positivist research to ensure rigour or trustworthiness. The process is one of checking with participants whether the information they have provided is exactly what they intended, so it provides dependability and confirmability of the data and the meanings made of it by the researcher. Since the observations and descriptions were drawn from video recordings of interactions where all permissions were provided under a separate Human Research Ethics approval (see Appendix 2), and the files were recorded in 2011, member checking to date has not been possible. It is, however, a possibility for further research.

Transferability

As explained in relation to the phenomenographic approach for the present investigation, thick, rich descriptions were generated from the video recordings to build a database consistent with the recommendations of Lincoln and Guba (1985) and also Patton (2002). Thick, rich description was needed so that judgements about the degree of fit or similarity could be made by others who may wish to apply part of all of the findings to another scenario.

Dependability and Confirmability

The most important of these concerns an audit trail including an independent audit by a scholar who understood the nature of the methodological approach adopted, and preferably a person with some relevant knowledge of the substantive field. The Independent Auditor's statement can be found at Appendix 5. Examination of the audit process results in a dependability judgement about an investigation. Examination of the product of the independent audit results in a confirmability statement.

To enhance those aspects of trustworthiness described above, the researcher necessarily remained self-reflexive in interpreting what was observed in the interactions between the teacher and student interactions in the video recordings. The balance between subjectivity and reflexivity is achieved in phenomenology by using the bracketing process as described by Van Manen (1997), wherein the researcher becomes "aware of one's own implicit assumptions and predispositions and sets them aside to avoid having them unduly influence the research" (Morrow, 2005, p. 254). Finally, the researcher must clearly and explicitly represent the findings in a believable, confirmable way, explaining why they are significant. According to Williams and Morrow (2009), the outcomes of the research must directly address the research questions and be discussed in relation to the existing literature.

The means by which the researcher reflected upon the descriptions of the Key Elements in this present investigation was through the use of a reflective journal, a section of which can be found at Appendix 6. Each teaching session and assessment interviews was viewed and reviewed by the researcher, so that, over time, a research journal was developed and maintained. The journal contained descriptive phenomenological reflections involving the behaviours of the participants, including verbal and non-verbal communication by the teachers and students during their interactions in the teaching sessions. These richly described observations involved examining systematically and listening attentively during many hours of interaction between the student and the teacher. The reflective journal was an essential element of criteria for credibility and potential transferability, together with dependability and confirmability.

3.7 Methodological Considerations

In the present investigation, there were some important methodological considerations to note when using video records as a data resource. The first consideration is the sheer volume of video data that needs to be comprehensively analysed and re-analysed. There are approximately 33 hours of video for analysis, representing 48 teaching sessions. The transcription and analysis of video recordings were extremely time-consuming.

Second, the participating teachers and students were selected by a recognised expert—the Director of the Mathematics Intervention Specialist Program (Wright et al., 2011). The basis for selecting a teacher-student dyad, as identified earlier, was that the relevant teacher was considered to be an effective teacher in that the relevant student was considered to have made very good progress in learning whole-number arithmetic. These judgements were made by the expert, based on more than 20 years working in the field. However, his judgement represented but one view of effective teaching, and arguably there are many different perspectives and definitions of effective teaching in the related literature (see, for example, Morgan, Dunn, Parry, & O'Reilly, 2004). It is beyond the scope of the present investigation to develop a definitive view of ‘effective teaching’ other than to declare that in this investigation, the definition was linked to measurable learning achievements in mathematics and in particular, the learning of whole-number arithmetic.

Finally, to protect the identities of the participants, all video files selected for this research were kept confidential. These materials will be kept securely for seven years at Southern Cross University, as specified in the NEAF application and approval (see Appendix 2). All the participating teachers and students in this research remain anonymous, so that all written information about the participants has been de-identified and re-coded at the time of analysis. Because this is a *post hoc* investigation, the participants were and remain unknown to the researcher and not identified in any way by the reporting of the methodology and findings. Therefore, while it would be useful to undertake member-checking with the participants to extend the present investigation and give voice to the participants, this procedure could not be carried out.

3.8 Concluding Remarks

This chapter has laid out a theoretical justification for the methodology used in this investigation. Phenomenology as a methodology has been described. A phenomenological approach to data collection and analysis has been explained. The operationalisation of the investigation has also been outlined, together with the criteria for rigour that have been implemented. Now attention is directed to developing an understanding of the essence of Key Elements in learning whole-number arithmetic by Years 3 and 4 students in one-to-one teaching interventions.

Chapter 4 – **Analysis of the Data**

There were two main purposes of the investigation. The first was to identify and illuminate the nature of Key Elements of one-to-one instruction in whole-number arithmetic, for Years 3 and 4 students. The second was to conceptualise a framework, which draws on how teachers use a specific cluster of Key Elements to achieve particular pedagogical goals.

As stated in Chapter 1, the context of the present investigation draws on aspects of the Mathematics Intervention Specialist Program (Wright et al., 2011). In this program, teachers provide intensive, one-to-one instruction to low-attaining Years 3 and 4 students. In all the implementations of the Mathematics Intervention Specialist Program, teachers routinely videotaped the pre- and post-assessment interviews and their teaching sessions. These videotaped records were used as the primary data source in the present investigation. The videotaped records of teaching sessions constitute a rich source of data for describing and understanding one-to-one teaching in whole-number arithmetic, as it occurred in Mathematics Intervention Specialist Program teaching sessions.

The video records were observed closely and intensively to examine "moment-by-moment organisation of the conduct of interaction" (Erickson, 1992, p. 203) in order to identify the Key Elements of one-to-one instruction which were defined in Chapter 1. This chapter reports on how the processes of observation and data analysis were undertaken in the present investigation. First, the data set is described. Second, the analytical framework for investigating the Key Elements, which involved a sequence of thirteen interacting, non-linear phases, is described in detail.

4.1 Describing the Data Set

The primary data source for this investigation consisted of six sets of video records of teaching sessions involving one-to-one instruction in whole-number arithmetic, together with assessment interviews. The assessment interviews and the teaching sessions were conducted by four teachers, Amilia, Ava, Emma and Sophia, and involved six students, Kate, Mia, Ella, Hannah, Chloe and Ben. These are not the real names of the teachers or students, but names given here to protect the identity of the participants as per ethical guidelines. As indicated in Figure 4.1, Amilia instructed Kate and Mia in separate one-to-one sessions. Ava and Emma instructed Ella and Hannah, respectively and Sophia instructed Chloe and Ben in separate one-to-one sessions.

Figure 4.1 Teacher-student dyads

Teacher: Amilia Ava Emma Sophia

Student: Kate Mia Ella Hannah Chloe Ben

As stated in Section 3.4.1 (Chapter 3), for each of the six teacher-student dyads, the data consisted of video recordings of 7 or 8 teaching sessions, each of 25-45 minutes' duration, conducted over a period of 12 weeks. For each dyad, there were also related video recordings of pre- and post-assessment interviews of each student by the teacher. Supplementary materials in the form of pre- and post-assessment interview analysis sheets and student screening tests (see Appendix 7) constituted additional data for the investigation.

Table 4.1 shows, for each teacher-student dyad, the number of teaching sessions selected for analysis, the duration of the selected teaching sessions and the duration of the pre- and post-assessment interview.

Table 4.1 Number and duration of teaching sessions selected

Teacher-Student	Number of teaching sessions	Duration of teaching sessions	Duration of Pre- and post-assessments
Amilia – Kate	09	236:07	70:15
Amilia – Mia	09	223:34	96:11
Ava – Ella	09	217:47	116:25
Emma – Hannah	07	205:12	110:55
Sophia – Chloe	07	225:24	85:13
Sophia - Ben	07	237:45	118:48

4.2 Analysis of Collected Data

As explained in Section 3.5 (Chapter 3), the processes of observation and data analysis used an analytical framework for investigating the Key Elements, which involved a sequence of thirteen interacting, non-linear phases. Each of these phases of the analytical framework is described in detail as follows.

4.2.1 Viewing Attentively the Video Recordings of the Teaching Sessions

Each video recording was viewed several times. This process was judged as giving the researcher familiarity with the content of the video recordings. With this in mind, the researcher viewed the recordings without intentionally imposing a specific, critical lens on the teaching and learning.

4.2.2 Transcribing the Video Recordings

For analytic purposes, an important step in phenomenological analysis of video data is to have the videos transcribed (Cobb & Whitenack, 1996; Hycner, 1999; Powell et al., 2003). The transcripts included the participants' utterances and actions that constituted the behaviours and activities that were taking place. The transcripts also included the details of the instructional settings and "as much as possible noting significant non-verbal and para-linguistic communications" (Hycner, 1999, p.144). Large margins on both sides of the transcripts were left for subsequent comments or notes during later stages of the data analysis.

The reasons for transcribing the data in this investigation are consistent with the reasons suggested by Powell et al. (2003, p. 422). First, the manufacture of the transcripts afforded the researcher opportunities for extended, considered deliberations of talks and noted gestures. Second, analysing teacher-student dialogues allowed the researcher to examine the transcripts with a view to what this revealed about mathematical meanings and the understanding of the teacher-student construct. Examination of the transcripts also allowed an indication of the relevant participants' body movements such as writings, sketching, eye contact, and so on. Third, for practical purposes, transcribing was able to disclose significant categories that the researcher might missed when viewing video recordings. Fourth, apart from presentation purposes, transcripts were very useful for providing evidence of findings in the form of scenarios of teaching as in narrative reports.

4.2.3 Summarising the Teaching Sessions

4.2.3.1 Summarising the Content of the Teaching Sessions

For analytical purposes, researchers are required not only to familiarise themselves with the content of the video recordings but also to observe its fine detail (Powell et al., 2003, p. 416). For this reason, during viewing of the video recordings and transcripts, brief summaries were made as a review of each teaching session. The summaries included teaching segments, learning domains, settings used and tasks presented. The duration of each teaching segment was marked for time counting and segment location purposes. The time-coded descriptions, teaching segments, learning domains, settings used and tasks presented are explained as follows.

- Time-coded descriptions referred to as duration times which were marked at the start and end of each teaching segment or learning domain. The time code allowed the researcher to locate quickly particular episodes of teaching or to detemine how much time each teacher-student dyad spent on each learning domain in the data set.

- A teaching segment refers to a part of a mathematics intervention teaching session where a teacher uses a particular setting or a collection of settings for instruction in a particular learning domain.
- The learning domains are considered to be large topics of the whole-number arithmetic content learned by students. This corresponds to the arithmetic content learned in the first four or five years of school. Four learning domains were the focus on in this investigation and brief description of each domain can be found at Appendix 8.
    - A-Number words and numerals;
    - B-Structuring numbers 1 to 20;
    - C-Conceptual place value; and,
    - D-Addition and subtraction to 100.
- A setting refers to a physical situation used by the teacher when posing an arithmetical task. Settings can be:
    - material (i.e., a physical situation), for example, collections of counters, numeral tracks, arithmetic racks, ten frames, etc.;
    - informal written, for example, empty number lines;
    - formal written, for example, addition cards, addition task in vertical format; or,
    - verbal.

The term ‘setting’ refers not only to the material, written or verbal statements but also to the ways in which these (material, written or verbal statements) are used in instruction and feature in students’ reasoning. Thus the term setting encompasses the often implicit features of instruction that arise during the pedagogical use of the setting.

- Tasks refer to arithmetic problems or questions that the teacher posed during teaching sessions.

An exemplar of a summary of a teaching session involving the Sophia-Ben dyad is given in Table 4.2.

Table 4.2 Summary of a teaching session—Sophia–Ben

Segment	Learning domain*	Setting	Tasks
Segment 1 (00:30 – 07:10) Counting by 1s in the range 100 to 1000	A	Numeral roll	Saying short Forward Number Words Sequences (FNWSs)/Backward Number Words Sequences (BNWSs) starting from X, say then see. Say the number after/before a given number (say then see), naming numerals under lids.
		Verbal only	Saying short FNWSs/BNWSs starting from X verbally.
Segment 2 (07:14 –12:35) Small doubles Five-plus	B	Verbal only	Verbal: given a double, say the sum.
		Arithmetic rack shown	Say upper row, lower row, altogether. Given 2 numbers: 5 and something, say the sum, and then check it on rack
		Verbal	Given a five-plus, say the sum
		Five-plus cards	Given a five-plus card, read the sum and say the answer.
Segment 3 (13:40 -19:45) Near doubles	B	Arithmetic rack	Build the sum on rack 3+4, 4+5, 2+3,.. say the answer
		Near double additive cards	Given a near double sum card, say the answer
		Verbal only	Given a near double sum, say the answer
Segment 4 (19:49 – 29:15) Incrementing/Decrementing by 1s, 10s and 100s (flexible switching units)	C	100 squares and strips shown	Building a big number by using 100 squares and strips of dots, e.g. 727
		100-squares and strips	Screened 100 squares and strips, after each increment/decrement, say the number
Segment 5 (29:18 – 50:10) Adding across a decuple	D	Bob cards shown/ screened	38+4, 27+5 solve the task by adding across a decuple, then illustrate it in the Empty number line
		Given 2-digit number and 1-digit number, solve task and record Verbal/ written	Solve tasks posed verbally, then do tasks in workbook by using Empty number line.

Note: Learning domains: A-Number words and numerals; B-Structuring numbers 1 to 20; C-Conceptual place value; D-Addition and subtraction to 100.

4.2.3.2 Calculating the Duration of Teaching for Each Teacher-student Dyad

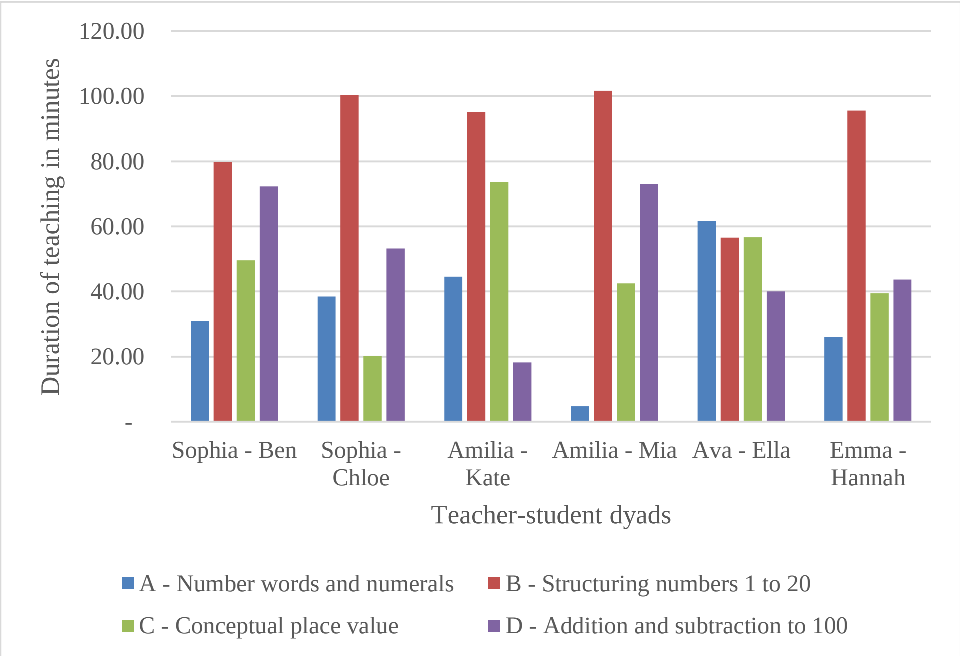
The researcher used time codes to measure the start and finish time in minutes and seconds of each teaching segment (in each teaching session) and these were noted in the summaries of the teaching session. The teacher then calculated the time durations that each teacher-student dyad spent on each learning domain. The durations of teaching across each learning domain for each teacher-student dyad are presented in Table 4.3 in minutes and represented graphically in Figures 4.2 and 4.3.

Table 4.3 Duration of teaching across each learning domain A-D, for each teacher-student dyad

Learning domains / Teacher-student dyads	A	B	C	D
Sophia - Ben	30.98	79.75	49.58	72.27
Sophia - Chloe	38.47	100.45	20.23	53.25
Amilia - Kate	44.6	95.18	73.55	18.22
Amilia - Mia	4.75	101.65	42.52	73.07
Ava - Ella	61.65	56.53	56.65	40.05
Emma - Hannah	26.12	95.63	39.42	43.72

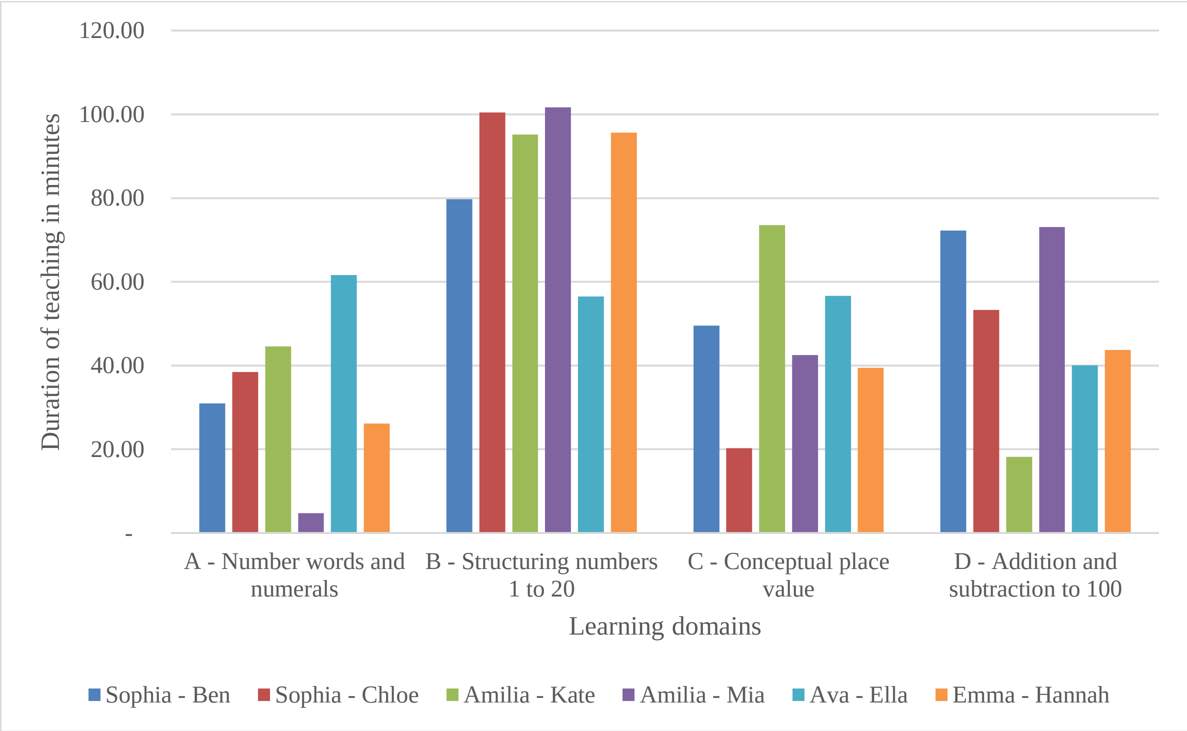
Note: Learning domains: A - Number words and numerals; B - Structuring numbers 1 to 20; C - Conceptual place value; and D - Addition and subtraction to 100.

Figure 4.2 Duration of teaching across each learning domain A-D, for each teacher-student dyad



In Figure 4.3, the data shown in Figure 4.2 have been re-arranged to show duration of teaching in the learning domains.

Figure 4.3 Duration of teaching across each teacher–student dyad, for each learning domain



4.2.4 Bracketing and Phenomenological Reduction

The phase of bracketing and phenomenological reduction followed observing the video recording of teaching sessions and reading the transcripts. The research data consisting of the video recordings and the transcriptions of the teaching sessions were approached with "an openness to whatever meanings emerged" (Hycner, 1999, p. 144). The following quote (Keen, 1975, p. 38 cited in Hycner, 1999, p. 144) explains and emphasises the significance of the phenomenological reduction in the data analysis process:

> The phenomenological reduction is a conscious, effortful, opening of ourselves to the phenomenon as a phenomenon. …We want not to see this event as an example of this or that theory that we have, we want to see it as a phenomenon in its right, with its own meaning and structure. Anybody can hear words that were spoken; to listen for the meaning as they eventually emerged from the phenomenon in its inherent meaningfulness. It is to have 'bracketed' our response to separate parts of the conversation and to have let the event emerge as a meaningful whole.

Thus the researcher is required to bracket as much as possible their meanings and interpretations from what has been observed and read from the research data. In other words, the researcher uses the matrices of their world view in order to understand the meaning of what has been observed and read, rather than what they expect to see (Hycner 1999, p. 144).

4.2.5 Observing the Video Recordings of the Teaching Sessions for a Sense of the Whole

This stage involves observing whole teaching sessions several times as well as reading the transcriptions of the teaching sessions. This is so the researcher can become familiar with the content of the teaching session as a whole. This provides a context for the emergence of specific units of meaning and themes afterward (Hycner, 1999, p. 145).

4.2.6 Delineating Units of General Meaning

Up to this point, the video recordings of the teaching sessions have been transcribed and a reasonably comprehensive knowledge of the content of the videos acquired by viewing and describing the video recordings through the previous phases. As well, the researcher's presuppositions have been bracketed consciously in order to stay as true to the data as possible. At this point the research questions were not yet addressed with respect to this data.

The next phase of the data analysis involved a rigorous process of observing closely the video recordings and going over words, expressions and sentences in the transcriptions in order to elicit the participants' meaning. Each teacher-student dyad was observed carefully throughout the video recording of teaching sessions and the transcriptions were carefully read in order to identify "units of general meaning" (Hycner 1999, p. 145). This process was carried out with as much open mindedness as possible in order to get at the nature of the meaning expressed in the literal data.

In the present investigation, a unit of general meaning is considered as an element which constitutes a story of what was going on in a teaching session. It involves describing the settings used, mathematical content and, as well, how the teacher and the student behaved in a particular instructional situation.

A brief illustration of the process is described in Figure 4.4. The context involves a scenario extracted from a teaching session of the Amilia-Mia dyad. This scenario focuses on addition by going through ten. The teacher, Amilia, used a setting with a dotted ten frame, counters and a screen.

Figure 4.4 Units of general meaning

Scenario	Units of general meaning
[1]Amilia: Look at this. (Places out a workbook). All I'm gonna do is give you eight. (Places out an 8-dot ten frame and an empty ten frame on table). Okay?	[1]Amilia asked Mia to look at the setting and got Mia's attention to the task that she is going to pose.
[2]Mia: (Nods)	[2]Mia showed her attention by nodding.
[3]Amilia: (Writes down the sum 8+5 in workbook) We're gonna do eight plus five. Okay? (Screens the ten frames) Want you to work that out in your head.	[3]Amilia posed a task of 'eight plus five', screened the ten frames and asked Mia to work it out in her head.
[4]Mia: Mm…	[4]Mia confirmed that she got the requirement.
[5]Amilia: Using that to build to ten. (Points at the sum in the workbook)	[5]Amilia then suggested a clue that might help to solve the task.
[6]Mia: (Immediately) I know it. (Grasps the pen)	[6]Mia immediately said she knows it.
[7]Amilia: What? (Looks at Mia)	[7]Amilia looked at Mia and provided wait-time while Mia attempted to solve the task
[8]Mia: It's… (After 12 seconds, writes '14' in the box)	[8]Mia was going to say the answer but then she stopped to think for 12 seconds before writing down the result, 14. [The answer was incorrect]
[9]Amilia: Okay. How'd you work it out?	[9]Amilia queried how Mia worked the problem out. [It was interesting that Amilia did not mention about the incorrectness of the answer.]
[10]Mia: Well, I remembered on the chart that there would have been, if there was five, one row. And I needed to add on one more which would have had to go up to there and that would add up to fourteen.	[10]Mia explained that she remembered on the chart she took one of the fives to add up to other addend to make fourteen.
[11]Amilia: Only one more? [12](Unscreens the ten frames). It's eight.	[11]Amilia queried Mia's explanation by asking "only one more?" [12]Amilia then unscreened the ten frames and said "it's eight".
[13]Mia: Oh. I thought it was nine.	[13]Mia realised her mistake when she saw it was eight but she thought it was nine.
[14]Amilia: So what's it gonna be?	[14]Amilia asked Mia "so what's it gonna be?"
[15]Mia: It's gonna be fifteen. (Looks at Amilia)	[15]Mia answered fifteen and looked at Amilia.
[16]Amilia: Fifteen?	[16]Amilia asked Mia "fifteen" with a high tone.
[17]Mia: (Immediately) No. Thirteen.	[17]Mia immediately corrected her answer to "thirteen".
[18]Amilia: There is another way you reckon? Thirteen. (Places five red counters on the empty ten frame). Let's have a look (points at the ten frames). You've got eight (indicates the 8-dot ten frame).	[18]Amilia asked Mia another way to solve the problem using the ten frames.
[19]Mia: Let's take up two.	[19]Mia suggested to take two red counters up.
[20]Amilia: Two up. (Moves the two red counters to the 8-dot ten frame)	[20]Amilia moved the two red counters to add to the 8-dot ten frame to make 10.
[21]Mia: That's thirteen.	[21]Mia said "That's thirteen".
[22]Amilia: Thirteen. Good girl.	[22]Amilia confirmed the correct answer and gave an affirmation to Mia.

4.2.7 Delineating Units of Meaning Relevant to the Research Question

Once the units of general meaning have been noted, the units of general meaning could now be addressed in light of the research questions. The units of general meaning were then reduced to units of meaning relevant to the research questions, called units of relevant meaning (Hycner, 1999, p. 146). This process obviously necessitated some sort of "judgment call" (Hycner, 1999, p. 147) on the part of the researcher. Therefore, the researcher needed to remain aware of bracketing presuppositions and the need for open mindedness with respect to the data.

A brief illustration of delineating units of meaning relevant to the research question is seen in Figure 4.5. The research question addressed to the units of general meaning (Figure 4.4) was as follows:

"What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?"

The definition of a Key Element was described in Section 1.3 (Chapter 1): A Key Element of one-to-one instruction is a micro-instructional strategy used by a teacher when interacting with a student in solving an arithmetical task. It is considered to be the smallest unit of analysis of teaching and has at least one of four functions involving the following.

- F1 – organising on-task activity;
- F2 – responding to student thinking or answering;
- F3 – adjusting task challenge within a task; and
- F4 – providing opportunities for students to gain intrinsic satisfaction from solving a task.

The units of relevant meaning are noted in Figure 4.5. They are the micro-instructional strategies used by Amilia when interacting with Mia in solving the additive task of 'eight plus five'. They have at least one of the four functions as noted in the second column of Figure 4.5.

Figure 4.5 Units of relevant meaning

Units of relevant meaning	Function unit is relevant to
[1]Amilia asked Mia to look at the setting and got Mia's attention to the task that she is going to pose.	[1]F1
[3]Amilia posed a task of 'eight plus five', screened the ten frames and asked Mia to work it out in her head.	[3]F1
[5]Amilia then suggested a clue that might help to solve the task.	[5]F1
[7]Amilia looked at Mia and provided wait-time while Mia attempted to solve the task	[7]F2
[9]Amilia queried how Mia worked the problem out. [It was interesting that Amilia did not mention about the incorrectness of the answer.]	[9]F2
[11]Amilia queried Mia's explanation by asking "only one more?" [12]Amilia then unscreened the ten frames and said "it's eight".	[11]F2 [12]F3
[14]Amilia asked Mia "so what's it gonna be?"	[14]F2
[16]Amilia asked Mia "fifteen" with a high tone.	[16]F2
[18]Amilia asked Mia another way to solve the problem using the ten frames.	[18]F2
[20]Amilia moved the two red counters to add to the 8-dot ten frame to make 10.	[20]F2
[22]Amilia confirmed the correct answer and gave an affirmation to Mia.	[22]F4

Up to this point, the original twenty-two units of general meaning have been reduced to twelve units of meaning relevant to the research questions with reference to the Key Elements of one-to-one instruction. The following section focuses on determining themes from the units of relevant meaning.

4.2.8 Determining Themes from the units of Relevant Meaning

At this stage, the units of relevant meaning had been examined to identify and label concepts or representations of teacher behaviours which have potential to be Key Elements of one-to-one instruction. In the present investigation, a concept was, in fact, an abstracted representation of a teacher behaviour that the researcher identified as being significant in the data. The concepts were labelled and developed by opening up the transcripts in NVivo and exposing the thoughts, ideas, and meanings contained therein (Strauss & Corbin, 1998). Labelling concepts allowed the grouping of similar teacher behaviours under a common heading or classification (Strauss & Corbin, 1998).

A brief illustration of determining themes from the units of relevant meaning is provided in Figure 4.6. The figure describes labelling teacher behaviours as abstract concepts from the data. Conceptual names are presented in the second column of the figure under the heading 'themes from the units of relevant meaning' as corresponding to the units of relevant meaning in the first column. For example, the unit of relevant meaning #1 was labelled 'pre-formulating a task' which refers to a statement and action by the teacher, prior to presenting a task to the student, that has the purpose of orienting the student's thinking to the coming task.

Figure 4.6 Determining themes from the units of relevant meaning

Units of relevant meaning	Themes from the units of relevant meaning
[#1]Amilia asked Mia to look at the setting and got Mia's attention to the task that she is going to pose.	[#1]Pre-formulating a task
[#3]Amilia posed a task of 'eight plus five', screened the ten frames and asked Mia to work it out in her head.	[#3]Screening
[#5]Amilia then suggested a clue that might help to solve the task.	[#5]Scaffolding before
[#7]Amilia looked at Mia and provided wait-time while Mia attempted to solve the task.	[#7]Post-task wait-time
[#9]Amilia queried how Mia worked the problem out. [It was interesting that Amilia did not mention about the incorrectness of the answer.]	[#9, 11, 14, 16]Querying an incorrect response
[#11]Amilia queried Mia's explanation by asking "only one more?" [#12]Amilia then unscreened the ten frames and said "it's eight".	[#12]Directing to check
[#14]Amilia asked Mia "so what's it gonna be?"	
[#16]Amilia asked Mia "fifteen" with a high tone.	
[#18]Amilia asked Mia another way to solve the problem using the ten frames.	[#18]Querying a correct response
[#20]Amilia moved the two red counters to add to the 8-dot ten frame to make 10.	
[#22]Amilia confirmed the correct answer and gave an affirmation to Mia.	[#22] Confirming and highlighting a correct response and affirming

During the process of determining themes from the units of relevant meaning, the identified themes were regularly brought into discussions with the researcher's supervisors, research students and teacher educators within the field of mathematics education. These discussions were an important part of the process, and feedback and disagreement resulted in changed names, changed or sharper definitions, and the merging and splitting of categories.

4.2.8.1 Using NVivo Software for Supporting Data Analysis

Determining themes from units of relevant meaning was not a step-by-step process. Instead, it was an iterative process where the researcher noticed, coded, took notes using memos, queried, and so on. NVivo 10 qualitative data processing software, therefore, was used to support analysing, managing, ordering, structuring, retrieving and visualising tasks. Each teacher-student dyad was analysed through intensive scrutiny to develop and refine categories related to the potential Key Elements. In that analytical process, the transcripts allowed the researcher to perform synchronous coding with the transcripts, while continually reviewing corresponding episodes of the video recordings by using two screens—one for coding with the transcripts inputted in NVivo and the other for viewing the videos. That process allowed the researcher to capture subtle nuances not only in speech but also non-verbal behaviours and visible patterns of behaviours. The process of analysing the data using NVivo in the current investigation was as suggested by Edhlund (2011). This process is outlined below.

Inputting. The transcriptions of the teaching sessions, the results of pre- and post-assessment interviews and field notes were inputted into NVivo sources.

Exploring. Two screens were used simultaneously during the analysis phases. One was used to open the NVivo program with the transcriptions. The other was used to open a video of a teaching session which corresponded to the transcript on the first screen. The teacher behaviours of interests were captured by viewing them carefully and repeatedly to ensure that the appropriate behaviours were validated through cross-checking procedures.

Coding. Coding in NVivo permitted the grouping of related concepts to be organised in nodes. An initial framework of nodes was developed continuously. This process involved the units of relevant meanings being identified during the analysis phases. This stage of analysis allowed an exploration of the more complex aspects of nodes (Bryman, 2008). The process involved initial nodes being moved, merged and renamed.

Querying. The researcher used the queries in the NVivo Toolkit to gather the results in each node and review them in one document. The researcher also used matrix coding queries to reflect the occurrence of each node/all nodes across teaching sessions, teacher-student dyads and learning domains.

Taking notes using memo. During the analysis phase, the researcher's insights were recorded where appropriate and those memos were used when writing up the thesis.

4.2.9 Clustering of Units of Relevant Meaning to Form 'Parent-themes'

Once the units of relevant meaning had been listed, the researcher once again bracketed presuppositions, staying as true as possible to the phenomena. The units of relevant meaning were examined in order to determine if any of them naturally clustered together, that is, whether "there seems to be some common theme or essence that unites several discrete units of relevant meaning" (Hycner, 1999, p. 150). Such an essence emerges during the process of rigorous examination of the unit of relevant meaning singly, as well as eliciting the essence of that unit of relevant meaning given the context.

A brief illustration of the process of clustering units of relevant meaning to form parent-themes can be seen in Figure 4.7.

Figure 4.7 Clusters of relevant meanings

I. *Draws the student's attention before posing a task*

#1 Amilia asked Mia to look at the setting and got Mia's attention to the task that she is going to pose.

II. *Gets the student to be engaged when posing a task*

#3 Amilia posed a task of 'eight plus five', screened the ten frames and asked Mia to work it out in her head.
#5 Amilia then suggested a clue that might help to solve the task.

III. *Provides support to the student during solving a task*

#7 Amilia looked at Mia and provided wait-time while Mia attempted to solve the task

#9 Amilia queried how Mia worked the problem out. [It was interesting that Amilia did not mention about the incorrectness of the answer.]

#11 Amilia queried Mia's explanation by asking "only one more?"

#12 Amilia then unscreened the ten frames and said "it's eight".

#14 Amilia asked Mia "so what's it gonna be?"

#16 Amilia asked Mia "fifteen" with a high tone.

#18 Amilia asked Mia another way to solve the problem using the ten frames.

#20 Amilia moved the two red counters to add to the 8-dot ten frame to make 10.

IV. *Gives feedback and affirmation after solving a task*

#22 Amilia confirmed the correct answer and gave affirmation to Mia.

In Figure 4.7, all the units of relevant meaning in Figure 4.6 have been used to form a cluster and each individual meaning has been interrogated to determine its "essence". For instance, the essence of the unit of relevant meaning #7 (Figure 4.6) was "looked at Mia and provided wait-time". This was considered as a sort of support—giving sufficient time for Mia to solve the task. In interrogating the units of relevant meaning #9, 11, 14, 16, Amilia queried Mia's solution when Mia answered incorrectly. These queries seem to be a sort of support, in which Amilia provided Mia an opportunity to realise the mistake in Mia's solution method. In examining the unit of relevant meaning #12, Amilia unscreened the ten frames and said "it's eight". This could be considered as a sort of support, in which Amilia indirectly assisted Mia by letting her see the setting that was not available at the time of initially solving the task.

In examining the units of relevant meaning #18 and #20, after Mia arrived at a correct answer, Amilia gauged Mia's knowledge by asking if there was another way to solve the task. The process of examining and interrogating the units of relevant meaning above has resulted in a cluster of the units involving #7, 9, 11, 12, 14, 16, 18, 20 as they all seem to cluster together under a heading of "provides support to the student during solving a task". In the process of forming themes, the researcher went back and forth to and from the video recordings of teaching sessions to the list of units of relevant meaning to derive clusters of appropriate meaning.

4.2.10 Determining Themes from Clusters of Relevant Meaning

Up to this point, all the clusters of relevant meaning have been interrogated to determine the central themes (if any) which express the essence of these clusters. In the illustration used in the previous phases, four clusters were listed in Figure 4.7 regarding instructional strategies that the teacher used during a period of interactive one-to-one instruction with the student in solving an arithmetical task. Figure 4.8 shows the correspondence.

Figure 4.8 Determining themes from clusters of relevant meaning

Clusters of relevant meanings	Stages of solving a task
I. Draws the student's attention before posing a task	I. Before posing a task
II. Gets the student to be engaged when posing a task	II. Posing a task
III. Provides support to the student during solving a task	III. During solving a task
IV. Gives feedback and affirmation after solving a task	IV. After solving a task

4.2.11 Identifying General and Unique Themes for all the Teacher-student Dyads

Once all the above phases had been repeated with all teaching sessions of each teacher-student dyad, the researcher began to look for themes common to most or all of the teacher-student dyads as well as the individual variations. This procedure required "the phenomenological viewpoint of eliciting essences as well as the acknowledgement of existential individual differences" (Hycner, 1999, p. 154). The themes from units of relevant meaning identified in Phase 7 (Section 4.2.7) were examined in the light of the definition of Key Element of one-to-one instruction in order to determine the Key Elements.

The first step was to look for the themes common to all or most of the teacher-student dyads. These common themes were grouped together indicating a general theme that emerged in most or all of the teacher-student dyads.

The second step was to look for the themes that were unique to a single teacher-student dyad or a minority of the teacher-student dyads. These individual variations were significant for discussing later in the sense of the use of the Key Elements across the participating teachers.

Table 4.4 shows the frequencies of Key Elements used across the participating teachers.

Table 4.4 Frequencies of Key Elements used across the participating teachers

	Amilia-Kate	Amilia-Mia	Ava-Ella	Emma-Hannah	Sophia-Ben	Sophia-Chloe	Total
Affirming	317	300	245	345	409	274	1,890
Screening, colour-coding and flashing	76	219	181	134	98	123	831
Directing to check	76	55	57	8	53	71	320
Querying a correct response	56	53	31	99	36	34	309
Scaffolding during	71	36	60	63	22	16	268
Post task - wait time	42	19	27	55	14	33	190
Recapitulating	41	20	14	37	34	30	176
Explaining	17	25	13	42	12	21	130
Pre-formulating a task	18	8	27	19	29	10	111
Confirming, highlighting and privileging a correct response	31	23	27	2	10	3	96
Re-posing the task	20	7	13	12	1	3	56
Querying an incorrect response	20	13	5	5	0	1	44
Stating a goal	2	2	11	0	10	9	34
Giving a meta-explanation	7	9	6	4	4	2	32
Changing the setting during solving	3	2	2	11	7	3	28
Scaffolding before	4	10	3	2	1	0	20
Focussed prompting	2	7	1	9	0	0	19
Rephrasing the task	2	1	3	2	0	0	8
Introducing a setting	2	1	3	0	0	0	6
Directly correcting a response	5	0	0	0	0	0	5
Giving encouragement to a partly or nearly correct response	0	2	0	1	2	0	5
Referring to an unseen setting	2	0	2	0	0	0	4
Linking settings	0	3	0	0	0	0	3
Reformulating a task	0	0	0	3	0	0	3
Directly demonstrating	1	0	0	0	1	0	2

4.2.12 Contextualisating Themes

Hycner (1999, p. 155) emphasised that, after general and unique themes have been noted, it is helpful to place these themes back within the overall contexts or horizons from which they emerged. An example of this is seen in Figure 4.9.

Figure 4.9 Contextualization of themes

Scenario	Key Elements
I. Before posing a task	
[#1]Amilia: Look at this. (Places out a workbook). All I'm gonna do is give you eight. (Places out an 8-dot ten frame and an empty ten frame on table). Okay?	[#1]Pre-formulating a task
[#2]Mia: (Nods)	
II. Posing a task	
[#3]Amilia: (Writes down the sum 8+5 in workbook) We're gonna do eight plus five. Okay? (Screens the ten frames) Want you to work that out in your head.	[#3]Screening
[#4]Mia: Mm…	
[#5]Amilia: Using that to build to ten. (Points at the sum on the workbook)	[#5]Scaffolding before
[#6]Mia: (Immediately) I know it. (Grasps the pen)	
III. During solving a task	
[#7]Amilia: What? (Looks at Mia)	[#7]Post-task wait-time
[#8]Mia: It's… (After 12 seconds, writes '14' in the box)	
[#9]Amilia: Okay. How'd you work it out?	[#9, 11, 14, 16]Querying an incorrect response
[#10]Mia: Well, I remembered on the chart that there would have been, if there was five, one row. And I needed to add on one more which would have had to go up to there and that would add up to fourteen.	
[#11]Amilia: Only one more? [12](Unscreens the ten frames). It's eight.	
[#13]Mia: Oh. I thought it was nine.	[#12]Directing to check
[#14]Amilia: So what's it gonna be?	
[#15]Mia: It's gonna be fifteen. (Looks at Amilia)	
[#16]Amilia: Fifteen?	
[#17]Mia: (Immediately) No. Thirteen.	
[#18]Amilia: There is another way you reckon? Thirteen. (Places five red counters on the empty ten frame). Let's have a look (points at the ten frames). You've got eight (indicates the 8-dot ten frame).	[#18]Querying a correct response
[#19]Mia: Let's take up two.	
[#20]Amilia: Two up. (Moves the two red counters to the 8-dot ten frame)	
[#21]Mia: That's thirteen.	
IV. After solving a task	
[#22]Amilia: Thirteen. Good girl.	[#22] Confirming and highlighting a correct response and affirming

4.2.13 Writing Composite Descriptions of the Key Elements Identified

After the identifying and coding processes, the next step was to compose descriptions of each Key Element identified. This process was regarded as "the result of making sense of the data with particular attention to identified codes" (Powell et al., 2003, p. 430). At this analytical phase, each teacher's identified codes were observed closely in order to discern the emerging and evolving narrative of the data. This process involved refining a collection of Key Elements used by the teachers, first coded and then interpreted, in order to provide insight into the interaction between a teacher and a student in a one-to-one instructional context.

In this phase, in order to construct a storyline, the researcher had to go back and forth examining the use of the Key Elements across teachers and also across learning domains. In practice, the researcher had accumulated the properties and dimensions of each Key Element during the process of identifying it for subsequent development of its comprehensive description. Those properties and dimensions drew on the use of that Key Element across all the teaching sessions and across all the participating teachers.

As an example, when developing a description of the Key Element of *querying a correct response*, in NVivo the researcher can access all the references coded for the Key Element across the teaching sessions and across the participating teachers. At this stage, this feature of NVivo was very useful for aggregating the properties and dimensions of each Key Element. Figure 4.10 shows the references coded for the Key Element of *querying a correct response*, extracted from the data in NVivo.

Based on the properties and dimensions of the Key Element of *querying a correct response* accumulated during the process of identifying and illuminating, as well as reviewing the references coded in Figure 4.10, the researcher developed a comprehensive description of the Key Element.

Figure 4.10 References coded for the Key Element of querying a correct response

References coded
<Internals\\Transcripts\\Amilia--Kate\\10 Amilia--Kate> - § 10 references coded [10.46% Coverage]
Reference 1 - 1.81% Coverage
Amilia: How do you know that's six?
Kate: Because with the one what had (riffles through cards on table) um, seven. Mm. Yeah. This one. It's like the same. This one. You're just taking away.
Amilia: Oh. You just take one black one away and make it orange. Good girl.
Reference 2 - 0.30% Coverage
Amilia: Five. How do you know it's five?
Kate: Because three and two make five.
<Internals\\Transcripts\\Amilia--Mia\\01 Amilia--Mia> - § 8 references coded [8.79% Coverage]
Reference 1 - 0.72% Coverage
Amilia: How do you know that there's 8 black?
Mia: Because there's 3 on the top and 5 on the bottom.
Amilia: OK, five and three make?
Mia: 8.
Reference 3 - 0.56% Coverage
Amilia: Good girl. How can you tell that there's 6 there?
Mia: Because 3 is on the top and 3 is on the bottom.

4.3 Concluding Remarks

This chapter reported in detail how the processes of observation and data analysis were carried out in the present investigation. These processes included describing the data set and specifying the analytical framework for investigating the Key Elements which involved a sequence of thirteen interacting, non-linear phases. Excerpts from the teaching sessions were used to illustrate how the Key Elements were identified and illuminated.

Chapter 5 – **Key Elements of Intensive, One-to-one Instruction**

Chapter 5 focuses on answering Research Question 1 paying attention to identifying and illuminating Key Elements used in intensive, one-to-one intervention teaching.

Research Question 1: *What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?*

As explained in Chapter 3, for each teacher-student dyad, the video recordings of teaching sessions were observed carefully and repeatedly. The processes of observation and data analysis used the analytical framework for investigating Key Elements involving a sequence of thirteen interacting, non-linear phases described in Section 3.7 (Chapter 3) and further elaborated in Section 4.2 (Chapter 4). This resulted in a collection of 25 Key Elements which were identified in the present investigation. These 25 Key Elements were presented in two sets, Set A and Set B. Set A involved 12 Key Elements based on the data obtained in the present investigation and taking into account the relevant research literature. Set B involved 13 novel Key Elements that emerged during the analysis phase of the investigation. The two sets take into account clusters of the Key Elements and are likely to be useful for future analyses of one-to-one instruction.

This chapter begins by presenting a set of 25 Key Elements, then provides comprehensive descriptions of the Key Elements identified. Examples of the Key Elements are illustrated by using excerpts from the Mathematics Intervention Specialist Program teaching sessions. In addition, problematic teacher behaviours associated with one-to-one instruction, identified during the data analysis phase of the present investigation, are presented.

5.1 A Collection of Key Elements

The collection of 25 Key Elements, which resulted from the processes of observation and data analysis, is presented in two sets, Set A and Set B as follows.

5.1.1 Revising the Key Elements in Relation to the Research Literature—Set A

Table 5.1 lists a set of Key Elements, called Set A. Set A involves 12 Key Elements derived from the research literature. These Key Elements were included in order to test their viability for future analyses of one-to-one instruction. Some properties and dimensions of these Key Elements have been described in the research literature, but, in Section 5.2, they are described comprehensively based on the data obtained in this investigation and in relation to the research literature.

Table 5.1 A revision of Key Elements in relation to the research literature

Set A
Directing to check
Affirming
Changing the setting during solving
Post-task wait-time
Introducing a setting
Pre-formulating a task
Reformulating a task
Screening, color-coding and flashing
Querying a correct response
Explaining
Scaffolding before
Scaffolding during

5.1.2 New Key Elements Arising from the Investigation—Set B

Set B involves 13 Key Elements which resulted from the second sub-process of identifying Key Elements of intensive, one-to-one instruction (Table 5.2). These Key Elements arose during the analysis phase of the current study and therefore are likely to be useful for future analyses of Key Elements.

Table 5.2 Key Elements arising during the current investigation

Set B
Recapitulating
Giving a meta-explanation
Confirming, highlighting and privileging a correct response
Re-posing the task
Rephrasing the task
Stating a goal
Querying an incorrect response
Focussed prompting
Giving encouragement to a partly or nearly correct response
Referring to an unseen setting
Linking settings
Directly demonstrating
Directly correcting a response

5.2 Descriptions of the Key Elements

Section 5.2.1 provides a comprehensive description and discussion of the origin of each of the 12 Key Elements in Set A. As mentioned earlier, these Key Elements were derived from the literature and they now are presented with a consideration of the form of their occurrence in the data and the definition of a Key Element established earlier in this investigation. A description of each Key Element is developed and compared in terms of their properties and dimensions.

Section 5.2.2 provides a comprehensive description and discussion of each of the 13 novel Key Elements in Set B which emerged during the data analysis phase of the present investigation.

Examples of the Key Elements generally are presented in Section 5.2.3. However, for some particular Key Elements, for example, *giving a meta-explanation*, *rephrasing the task*, *stating a goal*, and *focused prompting*, examples are presented after their descriptions.

5.2.1 Descriptions of the Key Elements in Set A (KE1 to KE12)

5.2.1.1 Directing to Check (KE1)

Across the teaching sessions, it was found that the action of the teachers directing the students to check their answer or their solution occurred frequently during interactive teaching. This appears to be a very similar process to what was called ‘child checking’ by Wright et al. (2002). *Directing to check* refers to a situation where the teacher assists the student indirectly by asking or allowing the student to check their last response. Examples of the typical language used by the teacher when considering this Key Element are: “Let’s check!”, “Let’s check if you were right?”, and “Let’s have a look!” Also, this is sometimes done by using informal language, such as “Would you like a little sneaky peak?”, or by simply giving a non-verbal sign for a student to check and then asking, “Are you right?”

In the present investigation, the Key Element of *directing to check* was found where the teacher responded to either a correct or an incorrect answer from the student. In the case where the student answered correctly, directing to check had the purpose of assuring the student that their solution was correct. In particular, when noticing that the student answered with a lack of certitude, the teacher did not comment on the correctness of the answer but asked the student to check their answer.

In the case that the student answered incorrectly, *directing to check* had the purpose of indirectly assisting the student to solve a task. Student checking in this way typically involves a resort to an easier or simpler strategy. In the case of additive and subtractive tasks, for example, this might involve counting a collection that was screened previously. In addition, checking might involve using a device such as a hundreds chart or a numeral roll that was not available at the time of initially solving the task.

Having a student check their own solution is important in one-to-one instruction. Through routinely being directed to check the solutions, students begin to develop a sense of the notion of verification in mathematics. Verification as used here refers to the general idea that solutions to mathematics problems often lend themselves to being checked or confirmed by a procedure different from that by which the student initially solved the problem (Wright et al., 2006, p. 35).

The Key Element of *directing to check* was evident twice in the excerpt below. This excerpt focuses on saying a number word after/before a given number. The tasks initially were posed verbally, then involved a setting of a numeral roll for checking the answer. During the first task, “What number comes after number seventeen?”, when the teacher, Amilia, noticed that the student, Kate, lacked certitude in her correct answer, she asked Kate to check the answer using the numeral roll. The situation occurred quite differently in the second task, “What comes before sixty?” Kate changed her mind from the initial correct answer to an incorrect answer when Amilia asked her to check. Amilia then kept scaffolding Kate to assist her in solving the task.

> Amilia: Kate, can you tell me what number comes after number seventeen?
>
> Kate: After… eight. No... Seventeen?
>
> Amilia: After seventeen.
>
> Kate: Eighteen.
>
> Amilia: Can you check?
>
> Kate: (Finds the number 18 on the numeral roll, looks at Amilia and smiles)
>
> Amilia: Are you right?
>
> Kate: Yep.
>
> Amilia-Kate: (They Hi-five) Yoo hoo!
>
> Amilia: Kate, what comes before sixty?
>
> Kate: …Fifty-nine.
>
> Amilia: Check.
>
> Kate: No, sixty was it?
>
> Amilia: Yes. What comes before sixty?
>
> Kate: Fifty. No, before or after?
>
> Amilia: Before fif…before sixty.

… … …

Kate: Fifty-nine.

Amilia: Is that what you said?

Kate: Yep.

Amilia: (They Hi-five) You’ve got to trust yourself miss. You know lots of things.

5.2.1.2 Affirming (KE2)

Affirming refers to statements or actions by the teacher which affirm effort or achievement on the part of the student and acknowledge that the student response is correct (Wright et al., 2002). This acknowledgement might take the form of overt feedback (Brophy & Good, 1986), which might range from brief head nods or some other kind of sign to indicate agreement, for example, through short affirmation statements such as “Yes!”, “Right!”, “Well done!”, “Good work!”, ‘Hi 5!’. *Affirming* often occurred after solving a task or during solving a task (after which the student progressed to solving the task).

Affirming can be omitted on occasions, for example, when solving answer-focused tasks where the teacher focuses on getting the student’s answer but the nature of the task is such that it cannot be elaborated in terms of a strategy. In this case, the teacher might simply move on to the next task if the previous question was answered correctly.

In a one-to-one instruction context, the teacher is likely to have many opportunities to affirm the student’s progress. *Affirming* occurred in all the scenarios presented in Section 5.2.3. This is evidence that appropriate and justified affirmation used by teachers has a positive effect on students’ solving of tasks.

5.2.1.3 Changing the Setting During Solving (KE3)

Changing the setting during solving refers to a deliberate action on the teacher’s part in changing a material setting during the period when the student is attempting to solve a task (Wright et al., 2002). This often occurs when the student apparently reaches an impasse, that is, when the teacher perceives that the student is unable to solve the task that they are currently attempting. In using the Key Element of *changing the setting during solving*, the teacher deliberately introduces new elements which, from the teacher’s perspective, can be linked to elements in the original setting. Thus, the intention on the teacher’s part is that the new elements enable the student to reconceptualise the current task and arrive at a solution which was not available to the student before the change of setting. These kinds of changes to a setting during the course of the student's problem-solving can serve to assist the student in arriving at a solution.

C*hanging the setting during solving* was evident in Scenario 5.2 (see Figure 5.10) twice. Sophia initially posed a task "What's a hundred less than a thousand and fifty?" verbally and provided wait-time for 10 seconds (*post-posing wait-time*). Ben answered incorrectly by saying "One hundred and fifty". Sophia provided wait-time for 16 seconds (*post-responding wait-time*). Ben asked her to repeat the task. Sophia repeated the task (*re-posing the task*). Ben nearly got to a correct answer. Sophia looked at Ben and smiled encouragingly and said "Nearly, I think you've. Nearly there." (*Giving encouragement to a partly or nearly correct response*). Ben appeared to reach an impasse. Sophia brought out the arrow card sheet and said to Ben, "Can you make one thousand and fifty? See what it looks like." Sophia, therefore, changed the setting from posing the task verbally to using the arrow card sheet. After which, Sophia kept providing support (*scaffolding during*), but Ben appeared to reach an impasse again. Sophia then decided to change the setting by using dot materials and help Ben to solve the task.

Changing the setting during solving a task might sometimes not result in a necessary insight or reconceptualisation on the part of the student. This often occurs because the student is not able to conceive of the links between the new and the old settings although these links might be very evident to the teacher.

5.2.1.4 Post-task Wait-time (KE4)

Post-task wait-time refers to the teacher behaviour of providing sufficient time after posing a task, that has the purpose of letting the student to think about and solve the task (Brophy & Good, 1986; Ewing, 2005). The key to providing sufficient time is to realise that typically during the wait-time the student is engaged in sustained and active thinking (Wright, Martland, Stafford, et al., 2006, p. 33). Studies of science instruction have shown higher student achievement when teachers wait for about three seconds after posing a task (Brophy & Good, 1986). However, in the present investigation, the participating students are considered to be low-attainers, so the student should be provided with relatively longer periods of time, say up to one minute or longer (Wright, Martland, Stafford, et al., 2006). Also, appropriate wait-time in combination with well-chosen tasks provides the basic ingredients for advancements in and reorganisation of a student's thinking (Wright, Martland, Stafford, et al., 2006, p. 33).

When the Key Elements were first described by Wright et al. (2002), *post-task wait-time* was used to refer to wait-time that occurred after the teacher posed the task and before the student answers. In the present investigation, the term *post-task wait-time* has been changed to two different terms: *post-posing wait-time* and *post-responding wait-time*. These two terms are described below.

Post-posing wait-time refers to wait-time that occurs after the teacher poses the task and before the student answers. The main purpose of this is to provide sufficient time for the student to think about and solve the task. *Post-responding wait-time* refers to wait-time that occurs after the student has answered, in which case the answer could be for the initially posed task or for a subtask that arose during solving the task.

Post-responding wait-time might occur when the student answers correctly, but also shows a lack of certitude about the correctness of their solution to the task. In this case, the given wait-time might help the student self-check or self-confirm their answer. In addition, *post-responding wait-time* was observed to occur more frequently when the student answers incorrectly. This situation can be categorised into two cases as follows: (i) after wait-time the student is able to self-correct the solution; and (ii) if not, the teacher might provide support to the student to solve the task.

Post-posing wait-time and *post-responding wait-time* were evident in Scenario 5.2 (see Figure 5.10). In Scenario 5.2, for example, the teacher, Sophia, initially posed a task, "What's a hundred less than a thousand and fifty?" and waited 10 seconds (*post-posing wait-time*). The student, Ben, answered incorrectly by saying, "One hundred and fifty". Sophia looked at Ben with an implicit signal that the answer was incorrect and provided Ben a wait-time of 16 seconds rather than comment on the correctness of the answer (*post-responding wait-time*). Sophia then restated the task, changed the material setting twice during solving the task, and provided scaffolding to support Ben to solve the task.

5.2.1.5 Introducing a Setting (KE5)

A range of settings were used by the teachers across the teaching sessions. In this study, 'setting' refers to a situation used by the teacher when posing an arithmetical task. Settings can be (a) material (i.e., a physical situation), for example, collections of counters, numeral tracks, arithmetic racks, ten frames, etc.; (b) informal written, for example, *Empty number line*; (c) formal written, for example, addition cards, addition task in vertical format; or (d) verbal. The term setting refers not only to the material, written or verbal statements, but also to the ways in which these (material, written or verbal statements) are used in instruction and feature in students' reasoning. Thus the term setting encompasses the often implicit features of instruction that arise during the pedagogical use of the setting.

Introducing a setting refers to a situation where a teacher initially introduces a setting to a student. When this occurs, it is important to undertake preliminary explanations and activities in order for the student to become familiar with the setting. Wright et al. (2002) suggested a procedure to introduce a new setting as follows. The teacher places the setting on the table and tells the student what it is called. The teacher then proceeds with a series of questions in order to reveal the student's initial sense of, and idea about, the setting. In this way the teacher is able to gain insight into the ways in which the student is likely to construe and think about the tasks presented using the setting.

In the excerpt below, the teacher, Ava, introduces the setting of a partitioned five frame. Ava initially engages the student, Ella, in discussing the frame. Ava then used a plank five frame and some green and red counters to pose a sequence of tasks which involves replacing one or more red counters with green counters.

> Ava: (Places a plank five frame card on table) Do you know what this is?
>
> Ella: Um, five squares
>
> Ava: That's exactly right and we call it a five frame. So we know that there's how many there?
>
> Ella: Five squares.
>
> Ava: Five, that's exactly right. I'm gonna give you some red and some green counters. (Places counters on table). Okay? So you can put some red and some green, whatever you like.
>
> Ella: (Places counters on the plank five frame: green, red, and green)
>
> Ava: Can we put all the greens up one end and all the red up the other end, though? (Points at the counters on the five frame)
>
> Ella: (Rearranges the counters: green, green, green, red, and red).
>
> Ava: Yeah. Good job. Great. So, how many greens have you got? (Points at the green counters)
>
> Ella: Three.
>
> Ava: And how many reds? (Points at the red counters)
>
> Ella: Two.
>
> Ava: And how many altogether?
>
> Ella: …

Ava: Remember, how many are in this? (Indicates the squares in the five frame)

Ella: Five.

Ava: We don't need to count, do we? We know that when it's all filled up it equals?

Ella: Five.

Ava: Five. So, we've got three and two makes?

Ella: Five.

Ava: Five. Okay, so that was three and two makes five. Can we do something different?

Ella: Um, we could do four and one.

Ava: Okay, can you show me how you would do that?

… …

The conversation continued with Ava showing different ways to make five on the five frame involving five and zero, zero and five, one and four, and so on.

5.2.1.6 Pre-formulating a Task (KE6)

Pre-formulating a task refers to a statement and action by the teacher, prior to presenting a task to the student, that has the purpose of orienting the student's thinking to the coming task (Cazden, 1986; McMahon, 1998, pp. 21-22). *Pre-formulating* thus draws the student's attention to the setting or directs the student's thinking to related tasks solved earlier in the teaching session or in an earlier session. *Pre-formulating* also has the purpose of laying a cognitive basis for a new task or sequence of tasks that the teacher intends to pose (Wright et al., 2002). *Pre-formulating* is evident in the excerpt below.

This excerpt focuses on the partitions of 10. The teacher, Amilia, used a workbook and pen. The tasks were presented in the form of 3---->10, for example, the student, Mia, was asked to fill a number above the arrow in order to add to 3 to make 10. Amilia used the Key Element of *pre-formulating* to start doing the task sequence.

Amilia: (Brings out a workbook). Let's have a look. I want you to do some writing this time.

Mia: Yay.

Amilia: What I'm gonna do, remember yesterday we did this? (Points at the written work from yesterday in the workbook, see Figure 5.1). You said one you need nine more to make ten. You said two - you need eight more to make ten.

Mia: (Nods)

Amilia: Okay. (Turns to a new page of the workbook) What I'm gonna do today is I'm gonna give you the first number and the arrow and the ten and I want you to put in what goes above the arrow to make ten. (Writes down in the workbook, 5----->10). Okay?

Figure 5.1 Partitions of 10 in written tasks

5.2.1.7 Reformulating a Task (KE7)

Reformulating a task refers to a statement or action by the teacher after presenting a task to a student, and before the student has solved the task (Cazden, 1986; McMahon, 1998, pp. 21-22). *Reformulating* has the purpose of refreshing the student's memory of some or all of the details of the task or providing the student with additional information about the task which is thought to be useful to the student. In Scenario 5.3 (see Figure 5.11), for example, after posing a task (nineteen plus four), the teacher, Emma, used the Key Element of *reformulating* to refresh Hanna's thinking about the fact that 19 is 1 away from 20. This highlights a strategy called *building up through tens* (also called *jump to the decuple*) (Wright et al., 2012) which could be used to solve the current task and which Hannah has already used.

An additional characteristic was identified with respect to the use of the Key Element of *reformulating a task*. That is, when realising the student has not understood, has misunderstood or has misconstrued a task, the teacher's responses could involve presenting again all or part of the task. The teacher might use a different way or different setting to represent the task with the purpose of helping the student fully understand the task.

5.2.1.8 Screening, Colour-Coding and Flashing (KE8)

Screening, colour-coding and flashing refer to techniques used in presenting tasks. These techniques are observed frequently in the *Structuring numbers 1 to 20* learning domain (Wright et al., 2006). Each of these techniques is described by Wright et al. (2006, pp. 34–35) as follows.

Screening refers to a technique used in the presentation of tasks where the teacher conceals the material setting from the student. *Screening* has the purpose of developing student thinking in the sense of moving from using concrete materials to more formal arithmetic. An additive task involving two screened collections, for example, is presented as follows. The teacher briefly displays and then screens the first collection which normally is the larger collection. The teacher then tells the student the number of counters in the collection. This is followed by similarly displaying and then screening a second collection, and then telling the student the number of counters in the second collection. The teacher then asks the student how many counters altogether.

Colour-coding refers to a technique used in presenting tasks where the teacher intends to highlight the partitions of a number such as 5 or 10, by using two contrasting colours, for example, red and green. *Colour-coding* has the purpose of highlighting the additive structure of numbers. Partitioned ten frames (see Figure 5.2) and partitioned five frames (see Figure 5.3) (Wright, Stanger, Stafford, & Martland, 2006) are well-known settings involving colour-coding.

Figure 5.2 Partitioned ten frames

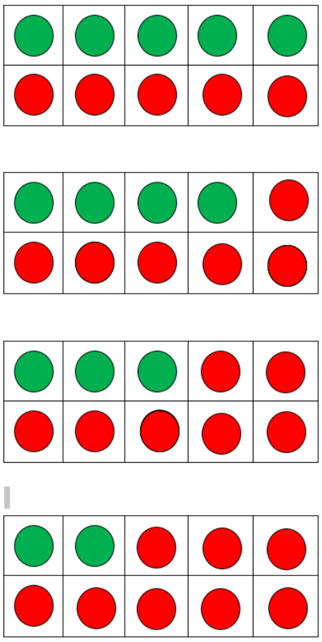
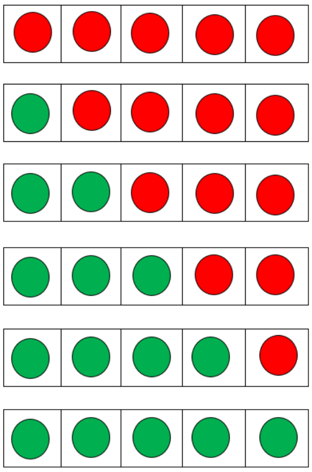


Figure 5.3 Partitioned five frames



One benefit of using *screening* and *colour-coding* in this way is that, during interactive teaching, the teacher can direct the student to check their solution. This will involve the student unscreening and using counting to check their solution. In the *missing addend task* (e.g., Wright et al., 2006), for example, which is presented using a different colour for each of the two addends, the student can check their solution by removing the screens.

Flashing refers to a technique used in presenting tasks which involve spatial patterns or settings for which spatial arrangement or colour-coding is particularly significant (Wright et al., 2002). The term *flash* is used in the sense of displaying briefly, typically for about half a second.

5.2.1.9 Querying a Correct Response (KE9)

Querying a correct response refers to situations where the student has responded correctly and the teacher questions the student about their response. Typically this will have the purpose of either helping to determine the student's solution method or gauging the student's certitude (Wright, 2010). Examples of the typical language used by the teacher when considering this KE are as follows: "How did you know that?"; "Tell me what you did?"; "Why do you think that is?"; "How do you know?"; "Show me how you did it?" or, "How did you work that out?" *Querying a correct response* is evident in Scenario 5.4 (see Figure 5.12). When the student, Kate, answered "Six" for the task of what is double three, the teacher, Amilia, queried Kate's answer by asking "How did you work that out?". Kate explained and showed that she used the counting-on strategy to solve the task. Amilia kept querying Kate by saying "Six. Good. The other way you could think about it is..." and then explained and supported Kate to use other ways to solve the task.

In the present investigation, it is observed that in some cases *querying a correct response* involves the teacher asking the student for another way to solve the task. The example described above in Scenario 5.4 is one such case. The teacher's action of asking the student for another way to solve the task could be regarded as a special case of querying a correct response.

5.2.1.10 Explaining (KE10)

Explaining refers to a situation where the teacher intends to engage the student in a conversation for the purpose of explaining some mathematical aspect or aspects relevant to the current instruction (Wright, 2010). The Key Element of *explaining* was evident in Scenario 5.3 (see Figure 5.11) and Scenario 5.4 (see Figure 5.12). In Scenario 5.3, for example, after solving the task, the teacher, Emma, explained to Hannah, how the strategy worked in solving the task by saying, "So see? When you make it up to a ten, it's just easy to add on, isn't it? When you make it up to one of these tens numbers" (points at the tens on arrow card sheet on table) "it's easy to add on".

Similar to the case of *querying a correct response*, in the present investigation, it is observed that when providing an explanation to the student after solving the task, the teacher sometimes comments with the purpose of evoking a different strategy for solving the task. In Scenario 5.4 (see Figure 5.12), when solving the task of 'double three', the student, Kate, initially used a counting-on strategy to get to the answer "3…, 4, 5, 6". After confirming Kate's answer, the teacher, Amilia, suggested to Kate another way to solve the task by using the previous double (double two) and counting by 2s.

5.2.1.11 Scaffolding Before (KE11)

Scaffolding refers to statements or actions on the part of the teacher to provide support for a student in an interactive teaching session (Wood, Bruner, & Ross, 1976, p. 90). For intensive, one-to-one instruction in particular, Wright (2010) categorised scaffolding into two main forms: s*caffolding befor*e and *scaffolding during*.

Scaffolding before refers to a situation where the teacher provides support prior to presenting the task or in the act of presenting the task. Thus *scaffolding* is referred to as *scaffolding before* in cases where the scaffolding is integral to the presentation of the task. In the excerpt below, the teacher, Amilia, showed Mia, the student, the partitioned ten frames. She then briefly described how she would use the ten frames in presenting tasks. This provided Mia with an initial orientation to the task to be posed.

> Amilia: Listen! I'm going to show you… what can you tell me about these (indicates the ten frame cards in her hand) with all the dots on them?

Mia: They've got 10 on them.

Amilia: They do. And how many on the top?

Mia: 5.

Amilia: And how many on the bottom?

Mia: 5.

Amilia: Beautiful.

5.2.1.12 Scaffolding During (KE12)

Scaffolding during refers to a situation where the teacher provides support in response to a student's unsuccessful attempt to solve a task. Thus this refers to scaffolding that is not provided during the presentation of the task. *Scaffolding during* is evident in Scenario 5.1 (see Figure 5.8), Scenario 5.2 (see Figure 5.10), and Scenario 5.5 (see Figure 5.14). In Scenario 5.5, for example, after posing a task—what comes before a hundred and eighty? The teacher, Ava, provided wait-time for 10 seconds (*post-posing wait-time*). The student, Ella, answered "One hundred and… ", and paused for six seconds. Ava provided support by using *scaffolding during*, she spoke to Ella "if we just think of it about, as eighty. What comes before eighty?".

5.2.2 Descriptions of the Key Elements in Set B (KE13 to KE25)

5.2.2.1 Recapitulating (KE13)

Recapitulating refers to a situation where the teacher reviews one or more strategies used while solving a task. This usually involves providing a brief summary of the process of how the task is solved. Such a review typically occurs after the student has solved the task. *Recapitulating* allows the teacher to emphasise crucial features of the student's strategy or solution with the purpose of providing an opportunity for the student to hear the teacher's account of their contribution to solving the task. In Scenario 5.1 (see Figure 5.8), for example, after supporting the student, Chloe, to solve the task—nine plus four— by using the *building up to tens* strategy, the teacher, Sophia, used the Key Element of *recapitulating* to summarise how the strategy was used to solve the task.

5.2.2.2 Giving a Meta-explanation (KE14)

Giving a meta-explanation refers to an explanation that is of a general nature rather than specifically related to tasks that the student is currently solving. *Giving a meta-explanation* typically takes the form of clarifying the meaning of a mathematical term, or describing the topic they are currently learning and a point to which the learning can progress. The excerpt below involves a conversation which happened after the student completed a sequence of tasks which focused on building patterns for numbers 11 to 20 on an arithmetic rack. The teacher, Sophia, described to the student, Ben, the usefulness of arithmetic racks in relation to other settings such as ten frames, with which Ben was more familiar.

> Sophia: Do you like using the rack?
>
> Ben: Mm... Hmm.
>
> Sophia: It's really good, isn't it? Do you think it will help you?
>
> Ben: Mm.
>
> Sophia: You know the way that you, you liked thinking about the dot cards, don't you? You know the ten frames. Do you think that will help you as well when you're adding and things?
>
> Ben: Mm hmm.
>
> Sophia: I think so, because you're good at imagining things.

The excerpt presented above exemplified the meta-explanation. Sophia's explanation was of general nature rather than being specifically related to tasks that Ben is currently solving. This typically takes the form of clarifying the meaning of a mathematical term and setting relevant to the current task.

5.2.2.3 Confirming, Highlighting and Privileging a Correct Response (KE15)

Confirming, highlighting and privileging a correct response refers to statements and actions by the teacher after the student answers correctly. This has the purpose of either (i) confirming the correctness of the answer, particularly in cases where the student appears to lack certitude, or (ii) highlighting and privileging the correctness of the answer in order to have the student reflect on their solution and thereby potentially increase their learning.

Confirming, highlighting and privileging a correct response is observed frequently in intensive, one-to-one instruction. This Key Element was evident in Scenario 5.5 (see Figure 5.14). After the student, Ella, answered correctly the task of writing the number just before 180, the teacher, Ava, used the Key Element of *confirming, highlighting and privileging a correct response* by saying "Right. So, before eighty is seventy-nine. So before one hundred and eighty must be one hundred and seventy-nine." This Key Element sometimes takes the form of confirming a correct response and then providing an affirmation. An example occurred in Scenario 5.4 (see Figure 5.12) where, after the student, Kate, answered correctly the task of double three, the teacher, Amilia, confirmed the answer by saying, "Six. Good!"

5.2.2.4 Re-posing the Task (KE16)

Re-posing the task refers to a situation where the teacher restates the task in order to help the student fully understand the task or to remind the student of some details of the task. In this situation, the student typically indicates that they cannot solve the task because they have lost track of some of the details of the task. In some cases the student explicitly requests a repeat of the task. This happens frequently in solving tasks that require incrementing and decrementing by ones, tens or hundreds, where the student forgets the result of the previous task. In this case, the student might ask "What number are we up to?" or "What was it again?"

Re-posing the task is observed frequently when the teacher responds to an incorrect answer from the student or when the student reaches an impasse. The Key Element of *re-posing the task* was evident in the Scenario 5.2 (see Figure 5.10). In Scenario 5.2, the student directly asked for repeating the task, she asked "what did you say again?"

5.2.2.5 Rephrasing the Task (KE17)

Rephrasing the task refers to the situation where the teacher expresses the task in an alternative way with the purpose of making the meaning clearer to the student. This occurs when the teacher judges that the student does not understand the words used by the teacher in relation to a mathematical aspect of the task. Thus the teacher changes their words in such a way that there is little or no change to the task. In the excerpt below, the teacher, Ava, rephrased the task by saying "take away ten", after first saying "counting backwards by tens" with which the student, Ella, was less familiar.

> Ava: Alright Ella, we're gonna do some counting backwards by tens. Okay?
>
> Ella: (Nods)
>
> Ava: So can you start at one hundred and ninety-three and count backwards by tens?
>
> Ella: One hundred and, one hundred and… eighty-nine.
>
> Ava: No, backward by tens. So one hundred and ninety-three take away a ten?
>
> Ella: One hundred and... Eighty-three.
>
> Ava: (Nods) Good girl.

5.2.2.6 Stating a Goal (KE18)

Stating a goal refers to a situation where the teacher summarises a student's recent progress and describes what needs to be practised more or what needs to be done next. This has the purpose of developing an action plan designed to motivate and guide the student towards a goal. The excerpt below describes a conversation between Amilia and Mia which occurred during practising the partitions of ten using a setting consisting of a block of 10 centicubes (see Figure 5.4), five of which were red and five of which were blue. The block can be segmented into two shorter blocks (e.g., 3 and 7). After doing a sequence of tasks focusing on partitions of 10, Amilia reviewed Mia's progress and concluded that they needed to practice partitions of 10 such as 7 and 3, and 8 and 2.

……

Amilia: 1 and 9 is an easy one, 5 and 5 is an easy one, 6 and 4 you're pretty good at. Maybe just 7 and 3 and 8 and 2 that you need to practise.

Mia: (Nods)

Amilia: (Breaks the block into a block of seven and a block of three. Puts the block of three on the table and hides the block of seven)

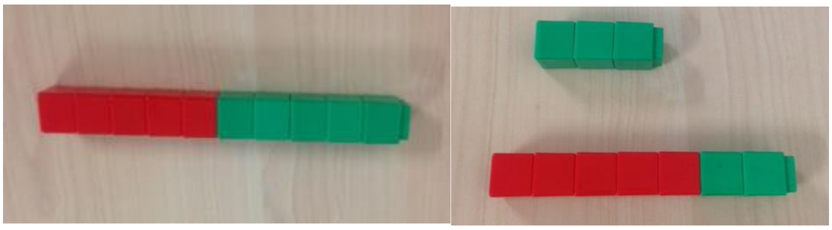
Mia: 3 and… 5? (Looks at Amilia)

Amilia: Is that?

Mia: (Immediately) 7.

….

Figure 5.4 A block of 10 centicubes



5.2.2.7 Querying an Incorrect Response (KE19)

Querying an incorrect response refers to a situation where the student answers incorrectly and the teacher questions the student about their response. Typically this has the purpose of (i) helping the student to realise the mistakes in their solution method on their own, so that the student might find a way to solve the problem or (ii) if the student still cannot self-correct, the teacher can then follow up by providing micro-adjusting or support to help the student to solve the task.

The excerpt below focuses on the task of incrementing by ten using bundling sticks. At this point the teacher, Amilia, has fifteen sticks under the screen. Kate's task is to say how many sticks there would be if Amilia adds another bundle of ten sticks. Kate initially answered "sixty-five". Instead of directly correcting Kate's answer, Amilia asks "Really? You don't look very sure." Kate then self-corrects. Amilia then directs Kate to check the answer by unscreening the sticks.

Amilia: Let’s go. Counting by tens. If there’s fifteen and I add ten. (Adds a bundle of 10).

Kate: Sixty-five.

Amilia: Really? You don’t look very sure.

Kate: …Twenty-five.

Amilia: (Unscreens the sticks). Is that better?

Kate: Uh ha.

In the following excerpt which involved the teacher, Sophia, and the student, Chloe, Chloe’s task was 9+9. Chloe initially answers incorrectly. The teacher, Sophia, queries Chloe’s answer by asking questions in relation to the answer and using scaffolding to help Chloe solve the task.

Sophia: (Places a 9+9 card on table)

Chloe: Nine plus nine is ... mmm ... (after 9 seconds) Nineteen …

Sophia: Well, nine plus ten is nineteen. Well would nine plus nine equals nineteen?

Chloe: No.

Sophia: Now, don't freak out when you see this one. What did you do for this one? (Places a 9+6 card on the table). What was your strategy for this one?

Chloe: Um, I took one away.

Sophia: Mm hmm. And made that...

Chloe: Fifteen.

Sophia: Okay, so can you do the same?

Chloe: Eighteen?

Sophia: Yeah. Don't freak out 'cos it's a double. Do the same thing. Build that up to ten, what have you got left to add on there?

Chloe: Nine? Umm... Eighteen?

Sophia: That's right, cos you're taking one from there to build that one up to ten and you got eighteen, yeah?

5.2.2.8 Focused Prompting (KE20)

Focused prompting has the purpose of asking, in an open-ended way, what the student is aware of or thinking of, for example: Is the student aware of an arithmetical pattern in a setting such as a sequence of partitions of 10 (9+1, 8+2, etc.)? (See Figure 5.5.)

Figure 5.5 Matching expression cards and partitioned ten frames

The excerpt below involves a conversation between the teacher, Amilia, and the student, Kate, when solving the task of matching expression cards and partitioned ten frames. At this point, they have matched each ten frame with the corresponding expression card (see Figure 5.5). Amilia asked Kate what she notices about the way that the partition and expression cards are organised.

> Amilia: You read me these, Kate. (Points at each expression card)
>
> Kate: Ten plus zero, one plus nine, two plus eight, three plus seven, four plus six, five plus five, six plus four, seven plus three, eight plus two, nine plus one, zero plus ten.
>
> Amilia: What do you notice about the way we organised them?
>
> Kate: (Points at each expression card and its corresponding partitioned ten frame to indicate the number of red and blue dots, respectively, on the card. The first number on the card corresponds to the number of red dots on the ten frame and the second number corresponds to the number of green dots). They're like five and five; four and six; three and seven; two and eight; one and nine; zero and ten.
>
> Amilia: Good. What happens as you go down the row this way? (Points to each of the partitioned ten frames in turn, on her right, from the top to the bottom)
>
> Kate: It's going by a pattern.
>
> Amilia: What's the pattern?
>
> Kate: (Points to the red dots on the ten frames) Zero, 1, 2, 3, 4, 5. It's going by ones.
>
> Amilia: If you come up this way – six… (Indicates the ten frames on her left from the bottom to the top). (Counts with Kate and points at the red dots on the ten frames when counting)
>
> Kate: 6, 7, 8, 9, 10.
>
> ……

5.2.2.9 Giving Encouragement to a Partly or Nearly Correct Response (KE21)

Giving encouragement to a partly or nearly correct response refers to situations where a student gives an incomplete or partly correct response. After this, a teacher usually would respond to indicate that the student is on track. This would involve confirming the correct part and then providing scaffolding. Concurrently, the teacher would encourage the student to continue, without being overly concerned about their inadequate response. Examples of the typical language used by the teacher when considering this Key Element are: “You are half way”; “You are almost there”; or “That is really good that you are thinking that”. This typically has a purpose of keeping the student on track and giving them more motivation and confidence to continue solving the task. The Key Element of *giving encouragement to a partly or nearly correct response* was evident in Scenario 5.2 (see Figure 5.10) when the student, Ben, was attempting to solve the task, and the teacher, Sophia, looked at him and smiled encouragingly and said, “Nearly, I think you’ve. Nearly there”.

5.2.2.10 Referring to an Unseen Setting (KE22)

Referring to an unseen setting refers to a situation where, when posing a task, the teacher reminds the student about a setting that has been distanced, that is, the setting was used at an earlier time in the teaching segment but is currently not being used. Referring to an unseen setting has the purpose of focusing the student’s thinking on how the teacher uses the setting when posing a task. In the excerpt below, in order to help the student, Kate, practise combining and partitioning 10, the teacher, Amilia, initially reminds Kate about the setting of partitioned ten frames.

> Amilia: Okay, we’ve been practising our tens facts. I’m going to show you some ten frames black dots, orange dots. Have we used these ones?
>
> Kate: Yep.
>
> Amilia: So I want to know how many black and how many orange. Okay? (Flashes a ten frame card)
>
> Kate: Seven blacks and three orange.
>
> ….

5.2.2.11 Linking Settings (KE23)

Linking settings refers to a situation where the teacher makes a connection between two or more settings. Linking settings has the purpose of enabling the student to regard an arithmetical problem from two or more perspectives. For example, base ten dot material (see Figure 5.6) could be linked to bundling sticks (see Figure 5.7), or a partitioned ten frame could be linked to an arithmetic rack. Figure 5.6 and Figure 5.7 show linking of the base ten dot material and bundling sticks to display the number 145.

Figure 5.6 Base ten dot materials

Figure 5.7 Bundling sticks

In the excerpt below, which involved the teacher, Amilia, and the student, Mia, focusing on a sequence of tasks of incrementing and decrementing by 1s, 10s and100s, and flexibly switching units, the teacher, Amilia, links the two settings of *plastic* and *bundling sticks*.

> Amilia: Okay. Now, last week we were doing counting by tens, both ways, forwards and backwards. (Brings out a bag of 100-dot squares and 10-dot strips). But we were using the bundling sticks. We're gonna do some higher numbers today. So the bundling sticks start to get a bit hard to use because there's so many of them.
>
> Amilia: Okay? So we use what we call the plastic. (Takes out some dot-strips). That's what this is. And it's called the plastic because it's made out of?
>
> Mia: Plastic.
>
> Amilia: You'd think they could come up with a better name for it. Okay. (Places a dot-strip on table) Have a look at that strip there. How many dots are there?
>
> Mia: Plastic sticks.
>
> Amilia: Plastic sticks. Maybe. Yeah.
>
> Amilia: How many dots are there on there?
>
> Mia: Ten.
>
> Amilia: There are ten. (Places more dot-strips on table) So every time you see one of these you know there's?
>
> Mia: Ten.
>
> Amilia: What do you reckon about one of these, then? (Takes out 100-squares)
>
> Mia: A hundred.
>
> Amilia: Good. Each one of these is worth a hundred.

Mia: Because there’s fifty and fifty.

Amilia: Good girl. There’s fifty on that side and there’s fifty on that side. Excellent!

5.2.2.12 Directly Demonstrating (KE24)

Directly demonstrating refers to a situation where, when commencing a new sequence of tasks, the teacher demonstrates how a task can be solved. This is similar to the practice in literacy instruction, of using a sequence of modelled, guided, and independent modes (Clay, 1979). Thus, *directly demonstrating* corresponds to modelling in literacy instruction. This Key Element did not occur frequently in the data collected in this investigation as the teachers were not encouraged to demonstrate to the student. Nevertheless, this Key Element shows its usefulness in some particular task sequences. One case of this is when a task involves a physical action by the student and the teacher models the action for the student.

The following excerpt which involved the teacher, Sophia, and the student, Ben, focused on building numbers in the range of 1 to 10 using an arithmetic rack. Sophia initially demonstrated how to use the arithmetic rack to build a number in quick movements.

Sophia: Okay, now we'll do some with the rack. (Brings out an arithmetic rack and puts it in front of Ben). Alright, so remember we were making numbers on the rack? (Indicates the beads on the rack).

Ben: Mm hmm

Sophia: Use the top row (points at the top row), then we used in pairs (slides a pair each from the right to the left with one bead on the top row and one bead on the lower row).

Ben: Mm hmm

Sophia: Use the top row (points at the top row), then we used pairs (slides one bead on the upper row and one bead on the lower row from right to left).

Ben: Mm hmm.

Sophia: Can you make five for me?

Ben: (Slides the five beads on the top row from the right to the left)

Sophia: Good. Nine?

… …

5.2.2.13 Directly Correcting a Response (KE25)

Directly correcting a response refers to a situation where the student responds incorrectly to the task. The teacher either (i) directly corrects the student’s response or (ii) directly indicates the student that they are incorrect and then directly corrects the student’s response. This can be useful in particular kinds of tasks such as answer-focused ones. Overuse of the Key Element could be counterproductive, particularly if it is used with a range of different kinds of tasks.

Directly correcting a response was evident in the following excerpt which focused on counting by tens and ones using bundle sticks. The teacher, Amilia, placed nine bundles of sticks on the table and asked Kate how many sticks. Kate answered incorrectly. Amilia then directly corrected Kate's answer.

> Amilia: (Places bundles and sticks on table) How many are there?
>
> Kate: Ninety bundles of ten.
>
> Amilia: No, not ninety bundles of ten. Ninety sticks, nine bundles of ten.
>
> Kate: Ninety sticks.

5.2.3 Examples of the Key Elements of One-to-one Instruction Used in Mathematics Intervention Specialist Program

Sections 5.2.1 and 5.2.2 provided descriptions and examples of the use of the Key Elements of one-to-one instruction. In this Section, excerpts from Mathematics Intervention Specialist Program teaching sessions are presented to illustrate the Key Elements. In the present investigation, these excerpts are referred to as rich scenarios and are extracted from videotaped records of Mathematics Intervention Specialist Program teaching sessions. Rich scenarios are characterised by a diversity of Key Elements. For each Key Element described in Set A or B, reference is made to one or more of the scenarios in which that Key Element is particularly evident.

Five scenarios of Mathematics Intervention Specialist Program teaching taken from the teaching sessions in the data set are now presented to illustrate the Key Elements of one-to-one instruction. The scenarios begin with an overview and serve to exemplify many of the Key Elements discussed in Set A and B of this section.

Scenario 5.1 (see Figure 5.8) involved the teacher, Sophia, and the student, Chloe. This scenario focuses on nine-pluses using the 'building up to tens' strategy. Sophia used a setting of an arithmetic rack and nine-plus cards.

Figure 5.8 Scenario 5.1 Sophia–Chloe

Scenario	Key Elements
Sophia: Okay. The other day we were having a look at …nine pluses, do you remember? What was our strategy for, umh (brings in an arithmetic rack) looking at nine-pluses. Let's say we had nine plus four. (Slides nine beads on the upper row and four beads on the lower row to the left of the arithmetic rack). (See Figure 5.9)	Pre-formulating a task
Chloe: Umm. You take away the ten, swap the four and it's thirteen. (Slides a left bead on the upper row to the left of the rack to make 10, also slides a bead of the four beads on the lower row to the right of the rack)	
Sophia: Okay. (Nods) That's right. So we're not really taking away the ten. (Slides the two beads that Chloe had just swapped back to where they were). We're … (holds the tenth bead on the upper row and pretends to slide it to the left of the rack to make 10). What do we call it?	Scaffolding during
Chloe: We're adding the ten and taking away the four.	
Sophia: Okay, so we're building up to ten, aren't we? Can you say that? (Slides back and forth the tenth bead)	
Chloe: We're building up to ten.	
Sophia: That's right. And then we use that bead to build up to ten. We need to move that one across. (Slides the bead on the lower row to the right of the rack).	Recapitulating

Figure 5.9 Using arithmetic rack in nine-pluses tasks

Scenario 5.2 (see Figure 5.10) involved the teacher, Sophia, and the student, Ben. This scenario focuses on decrementing by 100s. Sophia, initially posed a task verbally by asking ‘What's a hundred less than a thousand and fifty?’

Figure 5.10 Scenario 5.2 Sophia–Ben

Scenario	Key Elements
Sophia: What's a hundred less than a thousand and fifty? Ben: .... (After 10 seconds) One hundred and fifty.	Post-posing wait-time
Sophia: (Looks at Ben) Ben: No... (After 16 seconds) What did you say again?	Post-responding wait-time
Sophia: One thousand one hundred and fifty. Then a hundred less.	
Ben: (After 9 seconds). Three hundred and fifty? No. Ninety fi-, ninety f-, one hundred and, no, nine hundred and five. No. One hundred and five.	Re-posing the task
Sophia: (Looks at Ben and smiles encouragingly) Nearly, I think you've. Nearly there.	Giving encouragement to a partly or nearly correct response
Ben: What did you say it was...?	[Ben appeared to reach an impasse]
Sophia: So, it's one thousand. (Brings arrow card sheet in front). Can you make one thousand and fifty? See what it looks like.	Changing the setting during solving
Ben: (Makes up the number) Sophia: Now, a hundred less. Ben: … no hundreds in this Sophia: yes, so where could you take the hundred from?	Scaffolding during
Ben: Oh, the fifty? No. You take, you taking the hundred from a thousand? Sophia: Mm hmm. So how many is that? How many would I have left of that a thousand if I took a hundred away from it?	Scaffolding during
Ben: Fif-, no f-, five hundred. No. Sophia: Do you want to make it with the dots and see? Ben: Mmm. Sophia: Yep. (Gets plastics back out). One thousand and fifty, so you've got to make a thousand and fifty. Ben: (Lays out 100-squares on table) Sophia: Mm hmm. (Hands the ten-dot strips to Ben) Ben: (Lays out five 10-dot strips)	Changing the setting during solving
Sophia: Right, so how many have you got? How many dots? Ben: One thousand and fifty.	Scaffolding during
Sophia: Mm hmm. So you want a hundred less. Ben: (Takes one hundred-dot card away) Nine hundred and fifty.	Scaffolding during
Sophia: Good, Ben. Well done.	Affirming
Sophia: (Gets 1050 arrow card number). So, you had one thousand and fifty. Yeah?	Recapitulating
Ben: Mmm. Sophia: Where did you take the hundred from? Ben: From the one thousand. Sophia: Mm hmm. And when you took that one hundred away what did you have left? Ben: Nine thousand and fifty.	

Scenario	Key Elements
Sophia: Mmm…	
Ben: No, one hundred and fifty. Nine hundred and fifty.	
Sophia: Nine hundred and fifty. What does nine hundred look like?	
Ben: Umm	
Sophia: (Gets out arrow card sheet). There's nine hundred (points to it).	
Ben: (Takes 900 from sheet and starts to make up the number)	
Sophia: Then you ...	
Ben: Oh, fifty. (Grabs 50 arrow card).	
Sophia: That's it. Good on you. That's it. Well done. That's good.	Affirming

Scenario 5.3 (see Figure 5.11) involved the teacher, Emma, and the student, Hannah. This scenario focused on 2-digit addition by using building up through tens strategy.

Figure 5.11 Scenario 5.3 Emma – Hannah

Scenario	Key Elements
Emma: Okay. How many are there? (Places a 10-dot ten frame on table)	
Hannah: Ten.	
Emma: Okay. So if I put that there. (Puts a 9-dot ten frame beside the 10-dot frame). How many are there now?	
Hannah: Nineteen.	
Emma: Nineteen. Okay. Nineteen plus four. (Places a plain ten frame on table and puts four red counters on it). So this time, we're going to try to make it up to a ten. Because then we can have twenty.	Reformulating a task
Hannah: Twenty-four. Wait... (Puts one red counters from the 4 red counters on the 9-dot ten frame to make ten)	
Emma: Split the four. Good girl.	Affirming
Hannah: Twenty-three.	
Emma: Good girl. So see? When you make it up to a ten, it's just easy to add on. Isn't it?	Explaining
Hannah: (Nods)	
Emma: When you make it up to one of these tens numbers. (Points at the tens on arrow card sheet on table). It's easy to add on.	

Scenario 5.4 (see Figure 5.12) involved the teacher, Amilia, and the student, Kate. This scenario focuses on working with doubles. Amilia used a setting of arithmetic rack to present the doubles tasks and started with double one. At this point, Kate's task was double three.

Figure 5.12 Scenario 5.4 Amilia – Kate

Scenario	Key Elements
Amilia: (Slides six beads with three on the upper row and another three on the lower row from the left to the right of the rack) What if I've got double three?	
Kate: … (Looks at the rack for 5 seconds) Six.	Post-posing wait-time
Amilia: Good. How did you work that out?	Querying a correct response
Kate: (Points to the three beads on the upper row then counting the beads on the lower row) 3 ...4, 5, 6.	
Amilia: Six. Good. The other way you could think about it is... (Slides two beads (from the double three) with one on the upper row and one on the lower row to the middle of the rack). Double two is four and you've got two more?	Explaining
Kate: Six.	
Amilia: Do you wanna count in 2s? (Slides another two beads with one on the upper row and one on the lower row to the middle of the rack) I've got two … (Points at the two beads left on the right of the rack) (Slides back two beads from the middle with one on the upper row and one on the lower row of the rack to the right) then four. (Slides back another two beads from the middle with one on the upper row and one on the lower row to the right) then six. (See Figure 5.13)	

Figure 5.13 Using arithmetic rack in doubles tasks

Scenario 5.5 (see Figure 5.14) involved the teacher, Ava, and the student, Ella. This Scenario focused on solving a task of writing a number before a given number and involved using a workbook and a pen. Ella's task was to write the number just before 180.

Figure 5.14 Scenario 5.5 Ava – Ella

Scenario	Key Elements
Ava: (Writes a number on the workbook – 180). What's the number?	
Ella: A hundred and eighty.	
Ava: Mm. What comes before that?	
Ella: Um. … (After 10 seconds) One hundred and…(after 6 seconds)	Post-posing wait-time
Ava: If we just think of it about, as eighty. What comes before eighty?	
Ella: One hundred and seventy nine.	Scaffolding during
Ava: Right. So, before eighty is seventy nine. So before one hundred and eighty must be one hundred and seventy nine.	Confirming, highlighting and privileging a correct response

Table 5.3 Scenarios in which the Key Elements of one-to-one instruction are evident

No	Key elements	Scenarios				
		1	2	3	4	5
1	Directing to check					
2	Affirming		xxx	x		
3	Changing the setting during solving		xx			
4	Post-task wait-time		xx		x	x
5	Introducing a setting					
6	Pre-formulating a task	x				
7	Reformulating a task			x		
8	Screening, color-coding and flashing					
9	Querying a correct response				x	
10	Explaining			x	x	
11	Scaffolding before					
12	Scaffolding during	x	xxxx			x
13	Recapitulating	x	x			
14	Giving a meta-explanation					
15	Confirming, highlighting and privileging a correct response					x
16	Re-posing the task		x			
17	Rephrasing the task					
18	Stating a goal					
19	Querying an incorrect response					
20	Focussed prompting					
21	Giving encouragement to a partly or nearly correct response		x			
22	Referring to an unseen setting					
23	Linking settings					
24	Directly demonstrating					
25	Directly correcting a response					

Note: The “xx” in Table 5.3 indicates the number of times a Key Element occurs in each of the five scenarios.

5.3 Problematic Teacher Behaviours

Observing and analysing 48 teaching sessions in the data set provided significant insight into the good teaching practices of one-to-one instruction, that is, Key Elements. It also provided some insight into problematic behaviours associated with one-to-one instruction, for example, when a teacher provides unnecessary support or is unduly hasty. However, it is assumed that such elements would only be problematic if they were repeatedly visible in a teacher repertoire of strategies. Ten problematic teacher behaviours were identified during the data analysis phase of the present investigation and were categorised according to their occurrence in the following contexts, where the teacher is:

- presenting a task;
- providing support;
- giving an explanation; or,
- giving feedback.

These problematic teacher behaviours are presented in the table below.

Table 5.4 Problematic teacher behaviours

No.	Problematic teacher behaviours
1	Flagging a task as being difficult
2	Flagging a task as being easy
3	Simultaneously making more than one request
4	Interrupting the student
5	Inappropriately re-posing
6	Rushing or indecent haste
7	Miscuing
8	Red-herring
9	Non sequitur
10	Giving a ‘back-handed’ compliment

5.3.1 Presenting a Task

Flagging a task as being difficult is one example. When presenting a task, a teacher sometimes unintentionally raises the student’s anxiety about the coming task. This refers to a situation where, before presenting a task, the teacher advises the student that the coming task will be difficult or tricky. For example, the teacher says, “Are you ready for a super, super, super tricky one?” For some students, particularly those lacking confidence, such statements might make them think they are not going to be able to solve the task. This can hinder the student’s attempt to solve the task and reduce the student’s motivation.

Flagging a task as being easy is similar. It can also be counterproductive in terms of motivation in solving a task. This refers to a situation where, before presenting a task, the teacher advises the student that the coming task will be easy. For example, she says “Okay, now this is an easy one." Such a statement is likely to put additional pressure on the student to solve the task. If the student gives an incorrect answer, the student might feel uncomfortable about their ability and might lose confidence in solving tasks.

As well, when presenting a task, teachers sometimes confuse a student by making an unclear or unfocused request. A typical example is ***simultaneously making more than one request***. This refers to a situation where the teacher poses a task but, in doing so, asks the student an additional question which has the effect of confusing the student in that they do not know to which request to respond. This is illustrated in the following example which focuses on building patterns for numbers 1 to 10. The teacher, Sophia, used a setting of an arithmetic rack and a screen. The task sequence focused on quickly building patterns of pair-wise, 5-wise and 10-wise on the rack. In the excerpt, Sophia seems simultaneously to make at least three requests to Ben: (i) to tell him verbally how he is going to build a given number; (ii) to build that number on the rack; and (iii) to check his answer. Sophia's statements and actions seemed to confuse Ben and he responded incorrectly.

> Sophia: Okay. Now. (Places an arithmetic rack on table). I want you now... I'm going to say a number.
>
> Ben: Mm hmm.
>
> Sophia: And I want you to tell me how you're going to build the number here. (Indicates the rack)
>
> Ben: Mm hmm.
>
> Sophia: And then build it and check it.
>
> Ben: (Nods head)
>
> Sophia: Okay, so like if I say to you... say a number to me between one and ten.
>
> Ben: Five and four.

5.3.2 Providing Support

The teacher unnecessarily provides support such as *scaffolding*, *re-posing the task* or *rephrasing the task* when the student is solving the task. For example, the teacher provides scaffolding rather than wait-time. Such support might interfere with the student's thinking. The following examples are instances of providing unnecessary support that have been observed in the data set.

Interrupting the student refers to situations where the teacher distracts the student after they have already commenced solving a task. For example, the Key Element of *reformulating* is likely to be productive when the student seems genuinely to be unaware of critical information relating to the task, or requires additional information. However, when the student does not require a restatement of critical information or does not require additional information about the task, reformulating may be counterproductive. A counterproductive reformulation can distract the student and hinder their attempt to solve the task.

A second example, called ***inappropriately re-posing***, refers to a situation where the teacher unnecessarily re-poses a task, apparently to ensure that the student fully understands the task, but where, in fact, the student has already commenced solving the task. The following example illustrates this case where the task focused on incrementing by 100s. Note that in the following example, the inappropriate reposing was more apparent in the video record than in the transcript. The teacher, Ava, used a setting of dot materials including 100-squares, 10-strips and a screen. After Ava posed the task, the student, Ella, apparently commenced to solve the task. Nevertheless, Ava re-posed the task rather than providing wait-time for Ella. Re-posing in this way can be counterproductive because it might interrupt or distract the student.

> Ava: Okay, let's try this one. (Places out 100-squares and 10-strips on table) 100, 200, 300, 400, 500, 600 and nine. Happy with that?
>
> Ella: (Nods)
>
> Ava: (Screens all the 100-squares and 10-strips). Six hundred and nine. Add a hundred? (Adds a 100-square under the screen)
>
> Ella: Um…
>
> Ava: (After 6 seconds) Six hundred and nine.
>
> Ella: Seven hundred and nine.
>
> Ava: Good.

Another common problematic situation occurs when, in providing support to the student, the teacher proceeds unnecessarily quickly. This is referred to as ***rushing or indecent haste***. Thus, in this instructional situation, the teacher seems to be speaking and acting too quickly, and in some cases, this haste is transferred to the student. In any event, the haste on the part of the teacher is likely to be counterproductive.

Miscuing refers to a situation in which, after the student has commenced to solve the task, the teacher provides assistance to the student in the form of a hint or a suggested strategy, but in fact, the teacher's comment serves to mislead the student. An example occurs when the teacher directs the student's attention to a particular aspect of the setting or task, but the teacher's action serves to confuse the student.

Red-herring refers to a statement or action by the teacher when the student is solving a task, which results in the student being distracted or misled in their reasoning.

5.3.3 Giving an Explanation

Non sequitur refers to a statement by the teacher, which from the student's perspective, does not seem to logically follow on from or connect to the immediate prior discussion.

5.3.4 Giving Feedback

Giving feedback refers to a situation where the teacher gives a commentary on the student's solution such as highlighting aspects of the solution or recapitulating in the sense of giving a concise summary of the solution. The Key Element of affirming by providing feedback can be important in that it involves the teacher explicitly complimenting the student's effort or achievement. However, in some cases, feedback that is intended to provide affirmation can be counterproductive. One such problematic behaviour is ***giving a 'back-handed' compliment***. This refers to a situation in which the teacher compliments a student but in a way that tends to understate or underestimate the student's ability. An example is when, after the student has solved a task, the teacher says with a surprised tone, "That's great. I didn't think you would be able to do that."

5.4 Concluding Remarks

This chapter has presented the findings that emerged from the process of identifying Key Elements of one-to-one instruction. The focus of this chapter was to answer Research Question 1: '*What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?'* As well, this chapter addressed the problematic teacher behaviours that emerged from the data analysis phase.

Twenty-five Key Elements used during intensive, one-to-one instruction in the Mathematics Intervention Specialist Program were identified in the present investigation. These Key Elements were presented in Section 5.1 as two sets. Set A involved a revision of 12 Key Elements in relation to the research literature while Set B involved 13 novel Key Elements which emerged during the data analysis phase of the present investigation. The two sets take into account clusters of the Key Elements and are likely to be useful for future analyses of one-to-one instruction.

A comprehensive description of each Key Element of Sets A and B was provided in Sections 5.2.1 and 5.2.2 respectively. For each Key Element presented, excerpts from the Mathematics Intervention Specialist Program teaching sessions were presented to illustrate the Key Element.

Ten problematic teacher behaviours associated with one-to-one instruction were identified during the data analysis phase of the present investigation. Along with Key Elements identified and regarded as good teaching practice, the problematic teacher behaviours provided some insights into problematic behaviours associated with one-to-one instruction.

Chapter 6 – **A Framework for Analysing Intensive, One-to-one Instruction**

> The unravelling of the math lesson is a continuously reinvented process, with dozens of decision points at which the teacher moves on to the next activity format, which has only just emerged as a likely follow-on exercise, or switches to another exercise as a result of the drift of pupils' oral response, the level of pupils' task engagement, the time remaining until recess or the end of the period, or more likely, all these factors. This continuous readjustment results from what Lévi Strauss (1962) has called, felicitously, "engaging in a dialogue with the situation" as that situation unfolds. To tinker well here seems to depend on how quickly and accurately the teacher can read the situation. (Huberman, 1993, pp. 15-16)

The previous chapter examined individually, 25 Key Elements of one-to-one instruction. This chapter provides the context necessary for understanding how teachers use a specific cluster of Key Elements to achieve pedagogical goals, and how the Key Elements relate to each other, in intensive, one-to-one instruction. This chapter focuses on answering Research Question 2 as follows.

How can Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

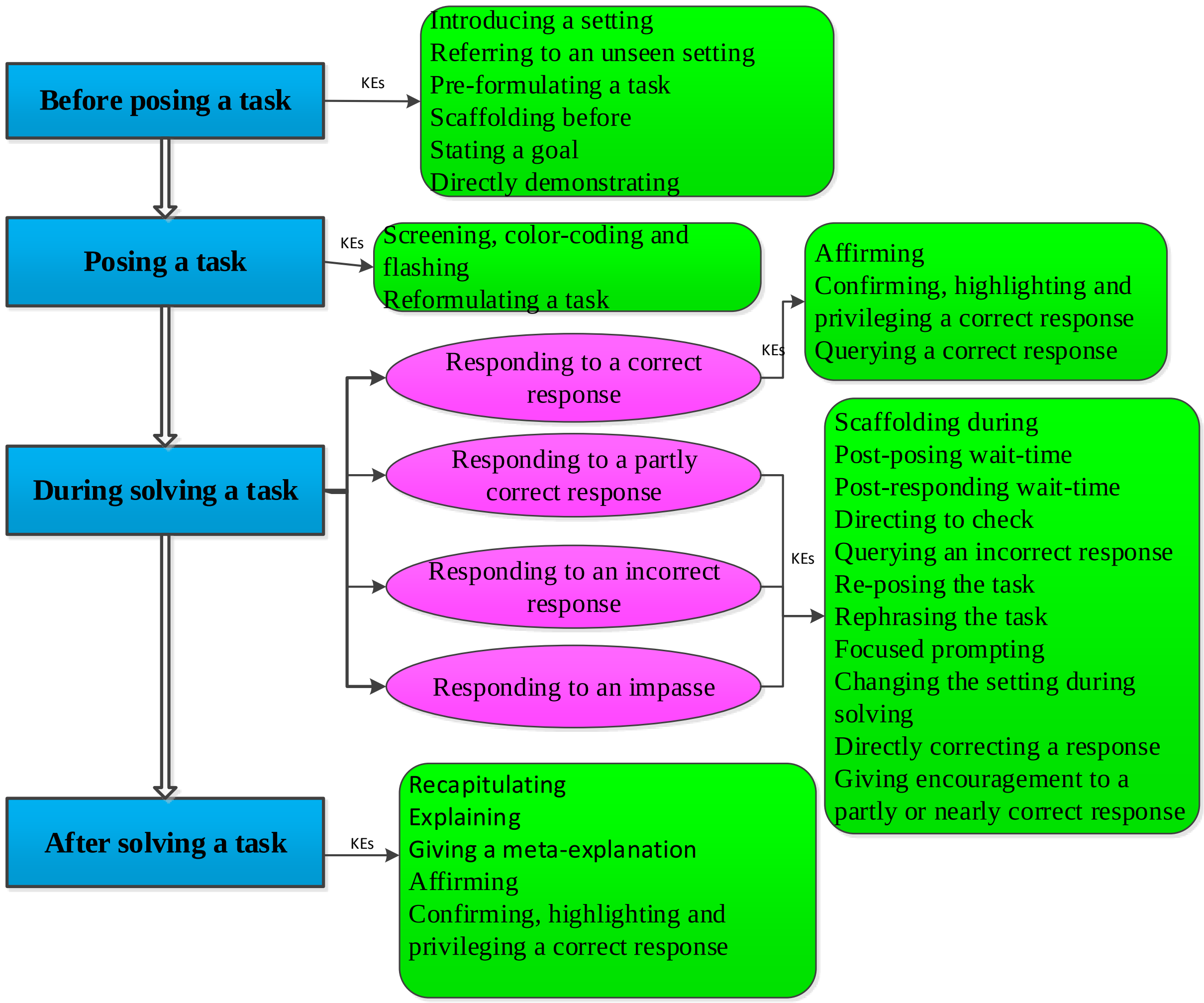
The chapter begins with a description of a conceptual framework for analysing intensive, one-to-one instruction in the Mathematics Intervention Specialist Program (Wright et al., 2011). The following section provides illustrations from Mathematics Intervention Specialist Program excerpts of the use of the framework which involves the four stages of solving an arithmetic task: before posing a task; posing a task; during solving a task; and after solving a task (see Figure 6.1). These excerpts were referred to as rich scenarios and extracted from videotaped records of Mathematics Intervention Specialist Program teaching sessions. Rich scenarios are characterised by a diversity of Key Elements.

6.1 Descriptions of the Framework

The conceptual framework for analysing one-to-one instruction that resulted from the analysis of the teacher-student interactions in the data is set out in Figure 6.1. The framework was developed to provide the context necessary for understanding how a teacher uses a specific cluster of Key Elements to achieve particular pedagogical goals.

The framework was layered into four stages of the teacher dealing with a task: A–Before posing a task; B–Posing a task; C–During solving a task; and, D–After solving a task. Collectively, these constitute the first or highest level of analysis. As well, the stage of C–During solving a task, is construed as four categories of teacher responses: C1–Responding to a correct response; C2–Responding to a partly correct response; C3–Responding to an incorrect response; and C4–Responding to an impasse. For each category, there are specific Key Elements that teachers used to respond to the students' responses. The four stages of this framework are presented by descriptions and discussions of illustrative excerpts from the Mathematics Intervention Specialist Program teaching sessions.

Figure 6.1 A framework for analysing one-to-one instruction



Note: KEs: Key Elements

6.1.1 Stage A—Before Posing a Task

Teachers typically intend to create a supportive environment for students before posing a task. It is important to undertake preliminary preparation of material settings and perhaps review mathematical knowledge in order for the students to be ready for the coming task. Statements and actions taken by the teacher before posing a task have the purpose of orienting the student's thinking to the coming task and drawing the student's attention to key features relating to the task setting. The teacher might tap related tasks solved earlier which leads to a connection with the coming task or might pose a problem that allows the teacher to direct the student's attention to the coming task. At this stage, prior to presenting the task, the teacher also determines the necessary support. The Key Elements typically used at the stage of before posing a task are *introducing a setting*, *referring to an unseen setting*, *pre-formulating a task*, *scaffolding before*, *stating a goal* and *directly demonstrating*.

6.1.2 Stage B—Posing a task

Teachers can present tasks involving material settings in several different ways. For example, when presenting tasks involving conceptual place value, the teacher might choose to display base ten materials. At a later point, the teacher might only momentarily display the material. Later still the teacher might choose to screen the material without displaying it. Varying the extent to which the teacher screens or displays the material exemplifies a particular dimension of mathematising called *distancing the setting* (Ellemor-Collins & Wright, 2011b, p. 135).

Necessarily *reformulating a task* is also a typical action taken by the teacher when posing a task. This involves the teacher realising that the student has not understood, has misunderstood or has misconstrued a task. The teacher's response could involve presenting again all or part of the task. The teacher might use a different way or different setting to present the task with the purpose of helping the student fully understand the task. Necessarily *reformulating* can involve simply *re-posing the task* or *rephrasing the task*. The Key Elements that are typically used at the stage of posing a task involve *screening, colour-coding and flashing* and *reformulating a task*.

Tasks presented by the teachers can be categorised as follows: answer-focused tasks; linked-tasks; and, strategy-focused tasks. Answer-focused tasks are tasks where the teacher focuses on getting the student's answer but the nature of the task is such that it cannot be elaborated in terms of a strategy (Munter, 2010). Linked tasks are tasks which link with the immediate prior task, in the sense that the answer for one task is used directly in the next task. Strategy-focused tasks refer to tasks where the teacher is interested in a particular strategy that the student uses to solve the task (Munter, 2010, p. 60).

6.1.3 Stage C—During solving a task

After a student initially responds to a task, the teacher's response is based on an evaluation of the student's mathematical understanding and strategy used. The teacher's response can be categorised into four cases as follows:

- Responding to a correct response;
- Responding to a partly correct response;

- Responding to an incorrect response; and,
- Responding to an impasse.

6.1.3.1 C1 – Responding to a correct response

Responding to a correct response refers to an instructional situation where the teacher responds to a correct answer from the student. The teacher's response takes account of the student's answer and typically has the purpose of extending and consolidating the student's understanding of the task and their solution. The Key Elements that are typically used in this case are *affirming*; *confirming, highlighting and privileging a correct response*; and *querying a correct response*. This results in actions by the teacher relevant to the task. The situation is described as follows. The teacher initially poses a task. The student responds correctly. The teacher's response to a C1 situation could be categorised as follows.

C1.1 The teacher gives affirmation and moves on to another task. This case occurs typically for answer-focused tasks. For example, consider a sequence of tasks during which the student states the number word before a given number word, the number word after a given number word, or a 'small doubles' task (e.g., 4+4). In particular, for some sequences of answer-focused tasks, after the student answers correctly the teacher moves quickly on to the next task, then gives affirmation at the end of a sequence of tasks.

C1.2 The teacher confirms, highlights and privileges the correct answer, and then gives affirmation. This case occurs typically for linked-tasks that is, tasks which link with the immediate prior task, in the sense that the answer for one task is used directly in the next task, for example, a sequence of tasks involving incrementing or decrementing a number by 1s, 10s or 100s and using bundling sticks or dot materials, unscreened or screened. For these tasks, after each increment or decrement, the student says the number. Therefore, confirming and highlighting a correct answer after each task helps the student to solve the next task.

C1.3 The teacher solicits the student's answer by asking the student to explain their strategy or thinking in solving the task. The teacher may ask the student to solve the task in a different way, for example, by using a different strategy (Munter, 2010, p. 48). Also, the teacher may encourage the student to examine the mathematical similarities and differences between two or more strategies. This case occurs typically for strategy-focused tasks referring to tasks where the teacher is interested in a particular strategy that the student uses to solve the task.

6.1.3.2 C2 – Responding to a partly correct response

Responding to a partly correct response refers to an instructional situation where a student gives an incomplete or partly correct response, after which the teacher responds to indicate that the student is on track by confirming the correct part and then follows up by providing scaffolding. Concurrently, the teacher encourages the student to continue without being overly concerned about their inadequate response. Examples of teachers' statements when using this Key Element are "You are half way" or "You are almost there".

6.1.3.3 C3 – Responding to an incorrect response

Responding to an incorrect response refers to an instructional situation where a teacher responds to an incorrect answer from the student. This results in actions by the teacher relevant to the task and typically has the purpose of helping the student to solve the task. The situation is described as follows. The teacher initially poses a task. The student responds incorrectly. The teacher's response to a C3 situation could be categorised as follows.

C3.1 The teacher responds by directly correcting the student's answer. This typically applies to *answer-focused* tasks.

C3.2 The teacher assists the student indirectly by asking or allowing the student to check their answer. Student checking in this way typically involves a resort to an easier or simpler strategy. Checking, therefore, might involve counting a collection that previously was screened or using a device such as a hundreds chart or a numeral roll that was not available at the time of initially solving the task.

C3.3 The teacher provides assistance which results in a less-challenging task. In this situation, the teacher typically uses one or more Key Elements such as *scaffolding during*, *post-task wait-time*, *querying an incorrect response*, *rephrasing the task*, *re-posing the task*, and *changing the setting during solving*. This typically applies to *strategy-focused* tasks.

6.1.3.4 C4 - Responding to an impasse

Responding to an impasse refers to an instructional situation where a teacher responds to a student who appears unable to solve a particular task at hand. In such situations, the teacher is likely to provide an appropriate adjustment or a scaffold for the student's learning. The situation is described as follows. The teacher initially poses a task. The student reaches an impasse. The teacher's response to a C4 situation could be categorised as follows.

C4.1 The teacher directly releases the student from the requirement to solve the task.

C4.2 The teacher tells the student the answer, then moves on.

C4.3 The teacher provides sufficient time for the student to be engaged in sustained and active thinking to solve the task. The student arrives at a method to solve the task. In this situation, the teacher typically uses the Key Element of *post-task wait-time*.

C4.4 The teacher micro-adjusts or provides scaffolding to such an extent that the student is now able to solve the task. In this situation, when necessary, the teacher uses a Key Element such as *scaffolding during*, *focused prompting*, *re-posing the task*, *rephrasing the task*, or *changing the setting during solving;* or a combination of some of those Key Elements to help the student to solve the task.

6.1.4 Stage D—After solving a task

After the task is solved, the teacher typically provides an opportunity for review and reflection. The student is engaged in a conversation for the purpose of explaining some mathematical aspect or aspects relevant to the current instruction. The teacher draws together what has been learned and summarises the key features of the student's strategies and insights. Eventually, success is celebrated. The Key Elements typically used at this stage involve *recapitulating*; *explaining*; *giving a meta-explanation*; *confirming; highlighting and privileging a correct response;* and *affirming*.

6.2 Illustration of the Use of the Framework

Section 6.1 provided a description of the framework for analysing intensive, one-to-one instruction. In this section, excerpts of teaching sessions from the Mathematics Intervention Specialist Program are presented to illustrate how the framework could be used. These excerpts are referred to as typical representative scenarios which correspond to the four stages of the framework.

6.2.1 Stage A—Before Posing a Task

An example of using the Key Element of *pre-formulating a task* at Stage A–Before posing a task, was evident in Scenario 5.1 (see Figure 5.8). In this scenario, before posing a task, the teacher, Sophia, reminded the student, Chloe, about the tasks of *nine-pluses* and the strategy used to solve such kinds of tasks that they had done on a previous day. This had the purpose of orienting Chloe's thinking to the tasks that they were going to do in the session. In that regard, Sophia used the Key Element of *pre-formulating a task*. Sophia spoke to Chloe as follows.

> Okay. The other day we were having a look at … nine pluses, do you remember? What was our strategy for, umh (Brings in an arithmetic rack) looking at nine-pluses. Let's say we had nine plus four. (Slides nine beads on the upper row and four beads on the lower row to the left of the arithmetic rack.)

An example of using the Key Element of *stating a goal* at Stage A–Before posing a task, is evident in the excerpt below. The excerpt involved the teacher, Sophia, and the student, Ben, working on *number words and numerals* tasks and involved a setting of a numeral roll and a multi-lid screen or numeral roll window (see Figure 6.2).

In the scenario, before posing a task to Ben, Sophia summarised Ben's recent progress in counting by indicating that Ben had done well in counting by tens. Sophia then set a goal for what they were going to do in the session, that is, count by ones with big numbers. Sophia also indicated that they would start counting with a numeral roll to keep track when counting (see Figure 6.2). Sophia showed a number on the numeral roll and Ben said the number forward from that number then Sophia revealed the number, and so on.

> Sophia: Okay. Now, you're going really well with your tens, so what we're going to have a go at today is counting a bit higher by ones.
> Ben: Mm hmm.
> Sophia: Okay. So we'll do it with counting things first and you will keep your track. (Gets out a numeral roll and a screen). (See Figure 6.2).
> … …

Figure 6.2 Using a numeral roll with a window

6.2.2 Stage B—Posing a Task

As stated earlier, the Key Elements that are typically used at the stage of posing a task involve *screening, colour-coding and flashing* and *reformulating a task*. The following excerpt illustrates how a teacher used the Key Elements of *screening* and *reformulating a task* when posing a task to a student.

The excerpt involved the teacher, Emma, and the student, Hannah, working on the tasks of incrementing and decrementing flexibly by tens and hundreds. In this episode, Emma used a setting of dot materials and a screen. At this point, Emma had six hundred and forty-nine which involved six 100-dot squares, four 10-dot strips and one 9-dot strip under the screen. The task for Hannah was to figure out how many dots would be under the screen if Emma added one more 10-dot strip under the screen.

Emma: (Adds a 10-dot strip under the screen)

Hannah: … (For 12 seconds)

Emma: Would you like to write it out?

Hannah: Yes.

Emma: Is it easier when you write it out? So, what were we up to? We were up to six hundred and forty-nine. (Places a small whiteboard and a texta pen on table) There you go. Six hundred and forty-nine is what you've got there.

Hannah: (Writes on whiteboard, 649)

Emma: Okay. Now I added ten, so what's ten more than that?

Hannah: (Immediately writes down the answer on the whiteboard, 659)

Emma: Good girl.

In the excerpt, Emma first posed the task by using the Key Element of *screening*. Emma used the screen to conceal the dot materials which displayed the number 649 with the purpose of developing student thinking in the sense of progressing from relying on concrete materials to not relying on concrete materials. After Emma posed the task, Hannah did not respond for 12 seconds, thus she apparently did not have an answer. Emma used the Key Element of *reformulating a task* by suggesting to Hannah that she writes the corresponding numeral. Thus Emma presented the task in written form rather than using screened dot cards.

6.2.3 Stage C—During Solving a Task

6.2.3.1 Responding to a Correct Response

Scenario 6.1 (see Figure 6.3) is an example of how a teacher responds to a student's correct answer. This corresponds to the case of C1.1 described in Section 6.1.3.1. In this case, after the student answers correctly, the teacher moves quickly on to the next task and gives affirmation at the end of a sequence of tasks.

The scenario involved the teacher, Ava, and the student, Ella working on tasks of doubles. The teacher gave a number verbally and the student said the double. These tasks are described as answer-focused. In the task sequence of "double ten", "double six", "double eight", "double nine", and "double seven", the teacher, Ava, simply moved on to the next task after Ella answered correctly and then gave affirmation after the last task of the sequence.

Figure 6.3 Scenario 6.1 Ava – Ella

Scenario	Key Elements
Ava: Ready for a quick quiz? Ella: (Nods) Ava: Double ten? Ella: Double ten is twenty. Ava: Double six? Ella: …Double… twelve. Ava: Double six is twelve. Double eight? Ella: Seven… sixteen. Ava: Double nine? Ella: Eighteen. Ava: Double seven? Ella: …Fourteen.	
Ava: Good girl. Well done. Beautiful!	Affirming

Scenario 6.2 (see Figure 6.4) is an example of how a teacher responds to a student's correct answer. This corresponds to the case of C1.2 described in Section 6.1.3.1. In this case, after the student answered correctly, the teacher confirmed, highlighted and privileged the correct answer, then gave affirmation. The scenario involved the teacher, Ava, and the student, Ella, working on the tasks of decrementing by 1s, 10s and 100s, and flexibly switching units, using screened dot materials. After each decrement, the student said the corresponding number. The scenario involved a setting of 100-dot squares, 10-dot strips and a 3-dot strip.

At the beginning of the scenario Ava had two thousand and forty three dots—20 squares, four 10-dot strips and one 3-dot strip under the screen. For each of the first three tasks Ava took away 10 in turn starting from two thousand and forty-three. Ella answered correctly. After each task, Ava quickly moved on to the next task. After the end of the sequence of tasks, Ava confirmed, highlighted and privileged Ella's answer by asking Ella to check her answer. Also, before moving to the next task, Ava repeated Ella's answer, for example, "Okay, two thousand, Ella". This might have helped Ella to solve the next task.

Figure 6.4 Scenario 6.2 Ava – Ella

Scenario	Key Elements
Ava: Okay, now, I'm gonna start taking things away, going backwards. (Screens the squares and strips) Right, one thousand…two thousand and forty-three take away a ten. (Takes away a 10-dot strip)	Screening
Ella: (Immediately) Two thousand and thirty-three.	
Ava: Take away another ten? (Takes away a 10-dot strip)	
Ella: (Immediately) Two thousand and twenty… three.	
Ava: Take away another ten? (Takes away a 10-dot strip)	
Ella: Two thousand and thirteen.	
Ava: Good girl. Take away another ten? (Takes away a 10-dot strip)	Affirming
Ella: … Two thousand and three.	
Ava: Take away three? (Takes away a 3-dot strip from under the screen)	
Ella: (Immediately) Two thousand.	
Ava: (Unscreens the squares and strips) Are you right?	Directing to check
Ella: Yeah.	
Ava: Okay, two thousand, Ella. Two thousand take away a hundred? (Takes away a 100 square from under the screen)	Confirms, highlights and privileges the correct answer
Ella: Um, one thousand and…and…one thousand nine hundred.	
Ava: Good girl.	Affirming

Scenario 6.3 (see Figure 6.5) is an example of how a teacher responded to a student's correct answer. This corresponds to the case of C1.3 described in Section 6.1.3.1. In this case, after the student answered correctly, the teacher solicited the student's answer by asking the student to explain their strategy or thinking in solving the task.

The scenario involved the teacher, Amilia, and the student, Mia working on the task of nine-pluses and involved using a workbook and a pen. The posed tasks are referred to as strategy-focused. Amilia, used the Key Element of *querying a correct response* to determine Mia's solution strategy and to gauge her certitude by asking how she worked out the problem.

Figure 6.5 Scenario 6.3 Amilia – Mia

Scenario	Key Elements
Amilia: Okay. Tell me what is nine plus six in your head? (Writes down a sum 9+6 in workbook) Mia: (Looks at the sum for three seconds) Fifteen. Amilia: How do you know? Mia: Well, because you said nine, you got a three and then you got another three it gets to fifteen. Amilia: Mmm… Is that the way you worked it out though? (Looks straight at Mia) Mia: (Nods) Amilia: So, what's nine plus three? Mia: Twelve. Amilia: Mm. Mia: Plus another three is fifteen. Amilia: Yes!	Querying a correct response

6.2.3.2 Responding to a partly correct response

Scenario 6.4 (see Figure 6.6) is an example of how a teacher responded to a student's partly correct answer. This example corresponds with the case of C2 described in Section 6.1.3.2. The scenario involved the teacher, Emma, and the student, Hannah working on the tasks of 2-digit subtraction without regrouping. The scenario involved using a whiteboard and texta pen and the setting of a numeral roll.

In the scenario, Emma initially posed the task by writing the task of 75-71 on the whiteboard. After four seconds, Hannah gave an incorrect answer by saying "Seventy four". Emma did not indicate that Hannah's answer was incorrect, but queried Hannah's answer and as well, re-posed the task in order to remind Hannah of some details of the task. Emma then encouraged Hannah by saying "You're half way." This served to keep Hannah on track and to give her more motivation and confidence to continue solving the task. The rest of the scenario used several other Key Elements including *giving encouragement to a partly or nearly correct response* to support Hannah in solving the task.

Figure 6.6 Scenario 6.4 Emma – Hannah

Scenario	Key Elements
Emma: What if I said this one to you (writes the subtractive task, 75 - 71 on the whiteboard), seventy five take away seventy-one?	
Hannah: (After 4 seconds) Seventy four. (Looks at Emma)	
Emma: Seventy-four? Did you say seventy-four? I've got seventy-five and I'm taking seventy-one away.	Querying an incorrect response, Re-posing the task
Hannah: …	
Emma: (After 5 seconds) You're half way.	Giving encouragement to a partly or nearly correct response
Hannah: Seventy three. (Looks at Emma)	
Emma: Look. (Places out the numeral roll) Where's seventy-five? And, where's seventy-one?	Changing the setting during solving
Hannah: Wait. There. (Points at the two numbers 75 and 71 on the numeral roll by using her two index fingers)	Scaffolding during
Emma: What's the difference?	
Hannah: 75, 74, 73, 72. (Points at each number in turn when counting). So, that's four.	
Emma: Did you…You said four? (Writes down number 4 as the result on the whiteboard) Before you said the answer was seventy four.	Explaining
Hannah: Oh. (Both laugh)	
Emma: It can't be seventy-four.	
Hannah: Yeah.	
Emma: Cause there's not seventy-four, is it? (Indicates the difference on the numeral roll)	
Hannah: No. (Both laughs)	
Emma: The difference is four.	

6.2.3.3 Responding to an incorrect response

Scenario 6.5 (see Figure 6.7) is an example of how a teacher responded to a student's incorrect answer. This example corresponds to the case of C3.1 described in Section 6.1.3.3. In this case, after the student answered incorrectly, the teacher responded by directly correcting the student's answer.

The scenario involved the teacher, Amilia, and the student, Kate working on a task sequence that involved saying forward number word sequences from a given number word. The scenario involved a setting of numeral tracks organised into four rows containing the numerals 8 to 47 (see Figure 6.8). The task took the form of saying the forward sequence from 24. Kate responded to the task by counting "Twenty-three, twenty-two…" Amilia directly corrected Kate by saying "No, forwards", and used her hand to show the direction on the numeral track. Kate then said the sequence correctly to 39 and then said '90'. Amilia uncovered the numerals

38, 39 and 40. Amelia and Kate chanted in unison ’38, 39, 40’. Kate continued until 50. In this scenario, Kate answered incorrectly twice.

Figure 6.7 Scenario 6.5 Amilia – Kate

Scenario	Key Elements
Amilia: I’m going to show you. (Opens a lid from a numeral track to show a number). That number! (See Figure 6.8)	
Kate: Twenty-four.	
Amilia: Good! Can you start counting forwards for me from there? (Indicates the number on the numeral track)	Affirming
Kate: Twenty-three, twenty two…	
Amilia: No, forwards. This way… (Points in the forward direction). Forward, twenty five…	Directly correcting student’s response
Kate: 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 90.	
Amilia: Oh…Hang on. Thirty-nine. We got to this bit (reveals the numeral 38 from the numeral track). Yep, thirty-eight, thirty-nine? (Reveals the numeral 39 and then 40).	Directing to check
Kate: 40, 41, 42, 43, 44, 45, 46, 47, 48, 49… 50.	
Amilia: Good girl. Well done! Okay.	Affirming

Figure 6.8 Using a numeral track

Scenario 6.6 (see Figure 6.9) is an example of how a teacher responded to a student’s incorrect answer. This example corresponds to the case of C3.2 described in Section 6.1.3.3. In this case, after the student answered incorrectly, the teacher assisted the student indirectly by asking or allowing the student to check their answer. The scenario involved the teacher, Amilia, and the student, Kate working on tasks which required saying a number that comes before or after several numbers (called jump around tasks). The scenario involved a setting of a numeral roll and a multi-lid screen or numeral window. This enabled Amilia to reveal a number on the numeral roll (see Figure 6.2). The task involved Amilia indicating a screened numeral and Kate figuring out what numeral Amilia had indicated.

In the scenario, Amilia indicated a screened numeral. After a period of eight seconds Kate answered incorrectly by saying “Ninety one”. Amilia did not indicate that Kate’s answer was wrong, instead she queried the answer by asking “Ninety one?” and then allowed Kate to open the lid and check her solution. Kate immediately corrected her answer. Amilia then confirmed, highlighted and privileged Kate’s correct answer by saying “Eighty-one. Because the eighties come after the seventies. Don’t they?”.

Figure 6.9 Scenario 6.6 Amilia – Kate

Scenario	Key elements
Amilia: Okay. What numbers that? (Points at the revealed number) Kate: Seventy eight. Amilia: Good. What’s this one under here? (Points at a covered number that is several numbers after 78’) Kate: … (After 8 seconds) Ninety one. Amilia: Ninety-one? You check. Kate: (Opens a lid to reveal the number) Eighty-one. (Looks at Amilia and smiles) Amilia: Eighty-one. Because the eighties come after the seventies. Don’t they? Okay.	Affirming Post-posing wait-time Querying an incorrect response, Directing to check Confirming, highlighting and privileging a correct response

Scenario 6.7 (see Figure 6.10) is an example of how a teacher responded to a student’s incorrect answer. This example corresponds to the case of C3.3 described in Section 6.1.3.3. In this case, after the student answered incorrectly, the teacher provided assistance. This is likely to result in a less-challenging task.

The scenario involved the teacher, Sophia, and the student, Chloe, working on the tasks of eight-pluses. Sophia first used an eight-plus card (8+2) for posing the task. Sophia’s teaching featured Key Elements to support Chloe in solving the task such as *post-posing wait-time*, *scaffolding during*, and *post-responding wait-time*. However, Chloe apparently reached an impasse. Sophia then changed the setting to one involving an arithmetic rack and continued to support Chloe. In doing so Sophia’s teaching featured the Key Elements of *scaffolding during*.

Figure 6.10 Scenario 6.7 Sophia – Chloe

Scenario	Key Elements
Sophia: Let's have a go at the cards. The eight-pluses. (Places an eight-plus card on table). Read that for me?	Post-posing wait-time
Chloe: Eight plus two. Mmm... (Counting on fingers surreptitiously, after 17 seconds). Sixteen? (Looks at Sophia)	
Sophia: There's a two in that one.	Scaffolding during
Chloe: (Looks at card again after 32 seconds) Fifteen? (Looks at Sophia)	Post-responding wait-time
Sophia: Okay. (Gets out the rack and places it on table) Work it out on here. Eight and two?	Changing the setting during solving
Chloe: (Slides eight beads on the upper row to the left, and two beads on the lower row to the left. Then slides the two remaining beads on the upper row to the left. Slides the two beads on the lower row to the right). Eight? (Looks at Sophia for 5 seconds)	
Sophia: (Looks at Chloe)	
Chloe: Eighteen?	
Sophia: How many beads are over here? (Points at the upper row)	Scaffolding during
Chloe: Ten?	
Sophia: What's eight and two?	
Chloe: Zero... (Looks at the upper row) Ten.	
Sophia: Ten. Do you see that? (Moves two beads to the right) How many beads are here? (Moves the last two beads on the upper row to the right and points at the other eight beads on the upper row)	Recapitulating
Chloe: Eight.	
Sophia: How many here? (Points at the other two beads on the upper row)	
Chloe: Two.	
Sophia: (Moves the two beads back to the left) When all the beads are across here? (Indicates the ten beads on the upper row)	
Chloe: Ten.	
Sophia: Ten.	
Chloe: (Laughs)	
Sophia: Yeah?	
Chloe: Yeah.	

6.2.3.4 Responding to an impasse

As described in Section 6.1.3.4, a teacher’s response to an impasse could be categorised into the four cases C4.1, C4.2, C4.3 and C4.4. In the case of C4.1, the teacher directly releases the student from the requirement to solve the task. In the case of C4.2, the teacher directly tells the student the answer, then moves on. The first two cases, therefore, are straightforward. In the case of C4.3, the teacher uses the Key Element of *post-task wait-time* (which is either *post-posing wait-time* or *post-responding wait-time*). This involves providing sufficient time for the student to be engaged in sustained and active thinking to solve the task, so that the student might arrive at a solution. The examples of the case C4.3 were evident in some of the Scenarios presented above, such as Scenario 6.6 (see Figure 6.9) and Scenario 6.7 (see Figure 6.10).

In case C4.4, when a student reaches an impasse, the teacher adjusts the task to make it less challenging and/or provides support to help the student solve the task. Scenario 6.8 (see Figure 6.11) is an example of case C4.4. The scenario involved the teacher, Ava, and the student, Ella working on the tasks of incrementing flexibly by ones, tens and hundreds. Ava used a setting of dot materials including 100-dot squares, 10-dot strips, a 7-dot strip and a screen. At the beginning of the scenario Ava has six 100-dot squares, nine 10-dot strips and a 7-dot strip under the screen. Ava then added a 10-dot strip under the screen. Ella's task was to figure out how many dots there were altogether.

In the scenario, Ava micro-adjusted and provided support to such an extent that Ella was able to solve the task. The Key Elements used to help Ella to solve the task included *post-posing wait-time*, *re-posing the task*, *scaffolding during*, *directing to check*, and *querying a correct response*. The use of these Key Elements in the scenario is described as follows.

After posing the task, Ava provided wait-time for Ella to think of the task by using the Key Element of *post-posing wait-time*. But, it seems that Ella could not give a complete answer and apparently reached an impasse. Ava then re-posed the task in order to remind Ella of the details of the task. After six seconds, Ella answered incorrectly by saying "Six hundred and seven". Ava provided scaffolding by removing the screen to reveal the dot materials, thus enabling Ella to see the dot materials. Ella still was not able to solve the problem. Ava then asked Ella to check by counting the squares and the strips. Ella then solved the problem by answering "seven hundred and seven".

Figure 6.11 Scenario 6.8 Ava – Ella

Scenario	Key Elements
Ava: Six hundred and ninety-seven. Add a ten? (Adds a 10-dot strip under the screen)	
Ella: Um… (After 5 seconds) Six hundred (looks at Ava)…six hundred (looks at Ava and smiles).	Post-posing wait-time
Ava: (After 4 seconds) Okay. Listen carefully. There was six hundred and ninety-seven and I added a ten. Six hundred and ninety-seven.	Re-posing the task
Ella: ... (After 6 seconds) Six hundred and seven.	
Ava: Let's have a look. (Unscreens the 100-dot squares and 10-dot strips). Six hundred (points at six 100-squares) and ninety-seven (points at the nine 10-dot strips and the 7-dot strip and picks up one 10-dot strip). That was there (referring to the prior collection of nine 10-dot strips). Then I added this ten. (Puts down the 10-dot strip)	Scaffolding during
Ella: Um…	
Ava: How many have we got?	
Ella: Um…	
Ava: You can check.	Directing to check
Ella: (Looks at the 100-squares and counts the 10-dot strips) One hundred.	
Ava: That's one hundred, is there?	Querying a correct response
Ella: Yes.	
Ava: Okay.	
Ella: So, seven hundred and seven.	
Ava: Good girl.	Affirming

6.2.4 Stage D—After solving a Task

As described in Section 6.1.4, after a task is solved, the teacher typically provides an opportunity for review and reflection. The Key Elements typically used at this stage involve *recapitulating*; *explaining*; *giving a meta-explanation*; *confirming, highlighting and privileging a correct response*; and *affirming*. Following are three scenarios illustrating the use of those Key Elements after the student has solved the task.

An example of using the Key Element of *recapitulating* was evident in Scenario 5.1 (see Figure 5.8) where the teacher, Sophia, summarized the strategy that was used in solving the task of 9 + 4 by using an arithmetic rack. Sophia said: "That's right. And then we use that bead to build up to ten. We need to move that one across. (Slides the bead on the lower row to the right of the rack)". (See also the picture in Figure 5.9).

An example of using the Key Element of *explaining* was evident in Scenario 6.4 (see Figure 6.6) where the teacher, Emma, explained to the student, Hannah, why her initial answer of

"Seventy four" was wrong. Emma said, "Did you … You said four?" (Writes down number 4 as the result on the whiteboard.) "Before you said the answer was seventy-four. It can't be seventy-four. Cause there's not seventy-four, is it?" (Indicates the difference on the numeral roll.) "The difference is four."

An example of using the Key Element of *giving a meta-explanation* after the task is solved can be seen in the excerpt used in Section 5.2.2.2. The Key Element of *confirming, highlighting and privileging a correct response* was evident in some presented scenarios such as Scenario 6.6 (see Figure 6.9). In the scenario, after the task was solved, the teacher, Amilia, confirmed, highlighted and privileged the correct answer by saying, "Eighty-one. Because the eighties come after the seventies. Don't they?"

6.3 Concluding Remarks

This chapter describes comprehensively the framework of Key Elements for analysing one-to-one instruction, outlines each stage of the framework and provides examples of use of the framework illustrated with excerpts from the Mathematics Intervention Specialist Program teaching sessions. This provides a comprehensive picture of how Key Elements feature in instruction involving interaction with a student in solving arithmetic tasks. The framework, therefore, could be used for teachers to reflect and refine their own teaching and for professional development purposes.

Chapter 7 – **Discussion**

Evans, Gruba and Zobel (2014, p. 11) describe a discussion chapter as a place where you:

> … critically examine your own results in the light of the previous state of the subjects as outlined in the background, and make judgments as to what has been learnt in your work.

This chapter is organised into four sections.

Section 7.1 includes a brief summary of the key findings of the investigation as well as an explanation of how valuable the findings are and why they are significant. The section suggests how the findings might be used and indicates who might use them.

Section 7.2 contains a discussion of the findings in relation to the broader educational and mathematical research literature. This involves an explanation of how the findings provide answers to the research questions and how these answers accommodate and add to previous research in the field.

Section 7.3 contains further discussion of the findings that focuses on the frequency of the Key Elements used by the participating teachers. As well, there is discussion about the expertise required to use the Key Elements, including teacher professional noticing and dimensions of mathematisation.

Section 7.4 provides some concluding remarks.

7.1 Key Findings of the Investigation

The present investigation focused on discovering, describing and illuminating the nature of Key Elements of one-to-one intervention teaching related to whole-number arithmetic with Years 3 and 4 students. The key findings of the investigation were twofold.

First, 25 Key Elements were identified in the teaching interventions, including the Key Elements that were already described in the research literature and novel Key Elements that emerged during the analysis phase of the investigation. The description of each Key Element in the context of teaching drew on the corpus of video recordings of the teaching sessions. As part of this identification and description, a set of problematic teacher behaviours was identified and described. These complement the collection of Key Elements in that they described problematic teaching practices associated with one-to-one instruction. Second, a framework of Key Elements for analysing one-to-one instruction was conceptualised. The framework provided the context necessary for understanding how teachers use specific clusters of Key Elements to achieve particular pedagogical goals.

These two findings arose from the two phases of the present investigation. In the first phase, Key Elements of intensive, one-to-one instruction were identified as used by a teacher when interacting with a student in solving an arithmetical task. In the second phase, a framework of Key Elements for analysing one-to-one instruction was conceptualised. Thus, while the first phase of this investigation examined the dialogue in expert tutoring on "the speech act level" (Cade et al., 2008, p. 1), that is, each Key Element was described individually; the second phase examined the tutorial dialogue as it occurred in a 'task block'. This provides the context necessary for understanding "how a series of speech acts relate to each other" (Cade et al., 2008, p. 1) and in this investigation it corresponds to how teachers use a specific cluster of Key Elements to achieve particular pedagogical goals.

7.1.1 Key Elements Identified and Their Significance and Implications

The first phase of identifying the Key Elements of intensive, one-to-one instruction resulted in a collection of 25 Key Elements that teachers used in interactive one-to-one teaching. In the collection, 12 Key Elements, listed as Set A of Key Elements in Section 5.1.1 (Chapter 5), were described comprehensively based on the data obtained in the current study and in relation to the research literature. As well, thirteen novel Key Elements emerged during the analysis phase of the current study and were described comprehensively. Collectively, the 25 Key Elements constitute a cluster of Key Elements likely to be useful for analysis of one-to-one instruction. It was intended that the comprehensive descriptions would provide a deep and richly layered understanding of the nature of the Key Elements. This understanding of the Key Elements would allow for extension and refinement of the research relevant to intensive intervention in whole-number learning.

The Key Elements could serve as a 'bank' of good teaching practices that teachers could use in their teaching. As described in Chapter 1, a Key Element of one-to-one instruction is a micro-instructional strategy used by a teacher when interacting with a student in solving an arithmetical task. A Key Element has at least one of four functions including organising on-task activity; responding to student thinking or answering; adjusting task challenge within a task; and providing opportunities for students to gain intrinsic satisfaction from solving a task. Thus, Key Elements collectively would cover almost all aspects of teacher-student interaction in a teaching session. Further, teachers could reflect on their own teaching when they are introduced to the collection of Key Elements and might well find that they have used many of the Key Elements prior to being introduced to them. Thus, they might well have used some of the Key Elements without knowing their names and without explicitly being aware that they were using them. They might also realise that they were aware of many aspects of the Key Elements without having an explicit understanding of them and without realising that they were using them. In other words, the teachers have used some of the Key Elements instinctively but not formally.

A study of teachers' beliefs about links between one-to-one teaching and classroom teaching (Tran & Wright, 2014a) found that intervention teachers were very positive towards transferring expertise, particularly the use of the Key Elements, developed in one-to-one teaching, to their classroom teaching. The participating teachers in Tran and Wright's study (2014a) were involved in both classroom and one-to-one intervention teaching, or at least they were formerly classroom teachers, therefore they were in a position to reflect on the expertise that they had developed in one-to-one instruction and were able to consider its application to classroom instruction and vice versa. In the context of classroom instruction, many conversations occur between the teacher and a student. Of course, other students in the class could also be involved in the discussion, but most just occur between the teacher and the student. Therefore, teacher-student dialogue in classroom and one-to-one contexts are very similar to each other and for this reason intervention teachers are likely to become aware of the usefulness of the Key Elements in classroom teaching.

The Key Elements of one-to-one instruction, therefore, are of practical importance because they are frequently observed in one-to-one intervention teaching. They are of theoretical importance because understanding them better can lead to more effective ways to characterise the range of instructional methods teachers use.

7.1.2 Problematic Teacher Behaviours Identified and Their Significance and Implications

A set of problematic teacher behaviours, as described in Section 5.3 (Chapter 5), is an additional outcome of the current study. Along with the collection of Key Elements regarded as good teaching practices, the set of problematic teacher behaviours provides insights into teacher behaviours associated with one-to-one instruction. During a teaching session, a teacher might unwittingly behave in a way that is regarded as problematic. These problematic behaviours include *flagging a task as being difficult, flagging a task as being easy, simultaneously making more than one request, interrupting the student, inappropriately re-posing, rushing or indecent haste, miscuing, red-herring, non sequitur* and *giving a 'back-handed' compliment.*

The teacher might not be consciously aware of their problematic teaching behaviour which might be a consequence of their regular teaching manner which in turn might be influenced by, for example, their teaching experiences, their mathematical content knowledge, their pedagogical content knowledge or the teaching environment. Making teachers explicitly aware of behaviours regarded as problematic could lead to a change in behaviour on the part of the teacher.

If the problematic teacher behaviours could be introduced to the teachers, there might be some interesting reactions from the teachers. They might show their ‘aha moment’ by exclaiming that they had used those teacher behaviours many times in their teaching without being aware of them. For example, teachers might realise that, in some cases, they had been too hasty or had provided unnecessary support that might well have interfered with the students’ thinking. Some teachers might realise that they had used flagging a task as being difficult and flagging a task as being easy quite frequently in their teaching when presenting a task to a student. Some teachers might assume that saying that the coming task is difficult would encourage the student and challenge them to pay more attention to the task; or when they say that the coming task will be easy that would result in less pressure on the student to solve the task. However, from the researcher’s point of view, those behaviours might result in negative effects on the students’ confidence and attitude towards mathematical learning. The concept of problematic behaviours could be interrogated from a perspective of frequency, though, technically, this matter was considered to be beyond the scope of this investigation.

7.1.3 The Framework for Analysing Intensive, One-to-one Instruction and Its Significance and Implications

The work on the second phase of the current study to develop a framework for analysing one-to-one instruction resulted in a comprehensive framework which provides a context for understanding how teachers use a specific cluster of Key Elements to achieve particular pedagogical goals. The framework of Key Elements could serve as a guide to leaders in mathematical instruction in their analysis of one-to-one instruction. Further, the framework could inform teachers working with low-attaining students in their professional practice by providing useful information about how teachers and students interact in mathematical interventions.

7.2 Discussion of the Findings in Relation to the Research Literature

7.2.1 In Relation to the Previous Research on Mathematics Recovery

One of the main focuses of the present investigation was to identify and illuminate Key Elements of intensive, one-to-one instruction focusing on whole-number arithmetic for Years 3 and 4 students. The investigation was a logical progression from previous studies on Year 1 students (McMahon, 1998; Wright, 2010; Wright et al., 2002) which involved investigating Mathematics Recovery intervention teaching. This section focuses on the differences and similarities in the findings of the present investigation compared with the findings of this earlier research.

As described in Section 5.2.1 (Chapter 5), a set of 12 Key Elements was identified, in the present investigation, that appeared to be similar to ones described by Wright et al. (2002) and Wright (2010). The description and discussion of each Key Element in relation to the research literature were presented with consideration of the form of their occurrence in the data of this investigation. The descriptions of these Key Elements presented in the present investigation focused on what, how, when and why the Key Elements were used. Some Key Elements, for example, *post-task wait-time* were reorganised into two different forms—*post-posing wait-time* and *post-responding wait-time*—in order to make clearer how, when and why each form was used. *Post-responding wait-time* could also have been placed in Set B of the Key Elements that emerged in the analysis phase but, for convenience in presenting the Key Elements, *post-responding wait-time* was presented as an expansion of *post-task wait-time*.

Concerning how to use a particular Key Element, the description of each Key Element presented in the current investigation usually involved some examples of typical statements or questions used by the teacher when using that Key Element. For example, typical questions asked when using the Key Element of *querying a correct response* were: “How did you know that?”, “Tell me what you did?”, “Why do you think that is?”, “How do you know?”, “Show me how you did it?” or “How did you work that out?” In the previous reports of Key Elements, examples of statements and questions were not provided in the descriptions of Key Elements. Providing these examples is regarded here as important for both teachers when using the Key Elements and educational leaders in analysing teaching.

Set B of 13 Key Elements described in Section 5.2.2 (Chapter 5) emerged from the analysis phase of the current investigation. That these did not emerge in the earlier studies may be attributed in part to the earlier studies focusing on teaching at Year 1 whereas the current study focused on teaching at Years 3 and 4.

Although some Key Elements of individualised teaching such as *micro-adjusting*, *teacher reflection* and *changing a task format* (Wright et al., 2002; Wright, 2010) were evident during the data analysis phase, these were not included in the collection of Key Elements in the present investigation. This is because, in the light of the definition of a Key Element, established in Section 1.2 (Chapter 1), a Key Element occurs within a task block, while *micro-adjusting*, *teacher reflection* and *changing a task format* occur across task blocks. Therefore they do not fit in the collection of Key Elements in the current study. However, these instructional strategies are significant in analysing one-to-one instruction as it occurs during a teaching segment or teaching session.

Some other Key Elements of individualised teaching such as *handling an impasse*, *responding to an incorrect response* and *responding to a correct response* (Wright et al., 2002; Wright, 2010) also were not included in the collection of Key Elements in this current investigation. This is because, from the researcher's point of view, *handling an impasse*, *responding to an incorrect response* and *responding to a correct response* seem more like instructional situations rather than instructional strategies used by the teachers. Therefore, in the present investigation, *handling an impasse*, *responding to an incorrect response* and *responding to a correct response* were determined to be nodes of the Key Elements rather than Key Elements per se. Each node corresponds to particular Key Elements as described in Figure 6.1 (framework for analysing one-to-one instruction) (Chapter 6). For example, the instructional situation of responding to a correct response includes Key Elements such as *querying a correct response*, *affirming*, and *confirming, highlighting and privileging a correct response*.

7.2.2 Relationships to Other Research on One-to-one Instruction

Decades of research on one-to-one instruction have furthered our understanding, to some extent, of the process of one-to-one instruction. However, the majority of this research has focused generally on non-expert tutors and provided little insight into the strategies that expert tutors use when interacting with students (Cade et al., 2008, p. 470; Lu et al., 2007, p. 456; Person et al., 2007). The present investigation is an in-depth analysis of instructional strategies used by expert tutors when interacting with students in solving arithmetical tasks. Table 7.1 describes some similarities among the Key Elements identified in the present investigation and the expert tutoring strategies described in the previous studies in Table 2.3 (Chapter 2). For example, expert tutoring strategies including *hint*, *prompt*, *pump* and *bridge* described by Person (2006) are specific cases of the Key Element of *scaffolding during*.

Table 7.1 Similarities among Key Elements identified and expert tutoring strategies in previous studies

Expert tutoring strategies described in the previous studies		Key Elements of one-to-one instruction identified in the present investigation
Expert tutoring strategy	**Reference**	
Goal-setting hints	Van Lehn et al. (2003)	Stating a goal
Explanation	Van Lehn et al. (2003)	Explaining
Hint Prompt Pump Bridge	Person (2006)	Scaffolding during
Summarise Re-voice	Person (2006)	Recapitulating
Ask clarification questions Ask comprehension-gauging questions	Person (2006)	Querying a correct response Querying an incorrect response
Give direct instruction Provide examples	Person (2006)	Directly correcting a response Directly demonstrating
Provide a preview	Person (2006)	Pre-formulating a task
Paraphrase	Person et al. (2007)	Rephrasing the task
Positive feedback Motivation/solidarity Attribution	Person et al. (2007)	Affirming Confirming, highlighting and privileging a correct response
Force a choice Provide counterexamples Complain	Person (2006)	*Have not been found in the present investigation*

7.2.3 Relationship to Sophisticated Tutoring Strategies

Regarding the sophisticated pedagogical tutoring strategies as described in Table 2.4 (Chapter 2), studies focusing on non-expert tutors found that the sophisticated pedagogical tutoring strategies referred to above are non-existent in their research corpus (e.g., Graesser et al., 1995, p. 502; Graesser et al., 1999, p. 39; Person & Graesser, 2003, p. 337). These researchers suggested that tutors need to be trained to be able to use the sophisticated tutoring strategies. The present investigation drew on expert tutors, therefore, it is worth examining the sophisticated tutoring strategies, if any, used by tutors in the present investigation when interacting with students in the course of teaching.

In the present investigation, the anatomy of expert tutors' teaching showed that expert tutors used the following sophisticated strategies: the Socratic Method, modelling-scaffolding-fading, and sophisticated motivational techniques. The following discussion shows how these were reflected in their use of the Key Elements described in this investigation.

7.2.3.1 Socratic Tutoring

Socratic tutoring is "a method of teaching based on asking a series of carefully constructed questions that would lead students to recognise and fix gaps and inconsistencies in what they know of a domain" (Collins, Warnock, Aeillo, & Miller, 1975 cited in Du Boulay & Luckin, 2001, p.236). In the Socratic Method, the teacher would neither indicate that the student's answer was wrong nor directly provide the correct answer or articulate the student's misconceptions (Graesser et al., 1995).

The teaching practices which involve some Key Elements such as *querying an incorrect response* and *post-responding wait-time* seem to be in alignment with Socratic pedagogical tutoring. In the use of the Key Element of *querying an incorrect response,* teachers question the student's response. In this way, teachers help the student to realise the mistakes in their solution methods on their own, following which the student might find a way to solve the problem. Otherwise teachers can then follow up by providing micro-adjusting or support to help the student to solve the task. Teachers' queries could provide opportunities for their students to express extended explanations and offer profound reasoning in answering the questions of why, how or what if. Teachers' queries offer great potential to encourage students to articulate lengthier answers which might display profound reasoning rather than rehearsing short snippets of trivial knowledge. Therefore, the Key Element of *post-responding wait-time* should be incorporated into the strategy of querying an incorrect response in order to provide sufficient time for students to recognise their mistake and, in some cases, to self-correct.

Scenario 7.1 (see Figure 7.1) describes a short tutorial dialogue between the teacher, Amilia, and the student, Mia focusing on addition by going through tens. Amilia uses a setting of dotted ten frames, counters and a screen. The scenario is followed by a discussion of the relation of the use of two Key Elements, *an incorrect response* and *post-responding wait-time,* and Socratic tutoring.

Figure 7.1 Scenario 7.1 Amilia—Mia

Scenario	Key Elements
[1]Amilia: Look at this. (Places out a workbook). All I'm gonna do is give you eight. (Places out an 8-dot ten frame and an empty ten frame on table). Okay?	
[2]Mia: (Nods)	
[3]Amilia: (Writes down the sum 8+5 in workbook) We're gonna do eight plus five. Okay? (Screens the ten frames) Want you to work that out in your head.	[3]Screening [3]Pre-formulating
[4]Mia: Mm…	
[5]Amilia: Using that to build to ten. (Points at the screened ten frames and looks at Mia) [Scaffolding before]	[5]Scaffolding before
[6]Mia: It's… (After 12 seconds, writes '14' in the box)	[6]Post-posing wait-time
[7]Amilia: Okay. How'd you work it out?	[7]Querying an incorrect response
[8]Mia: Well, I remembered on the chart that there would have been, if there was five, one row.	
[9]Amilia: Mm hmm.	
[10]Mia: And I needed to add on one more which would have had to go up to there and that would add up to fourteen.	
[11]Amilia: Only one more? (Unscreens the ten frames) [Directing to check] It's eight.	[11]Directing to check [11]Querying an incorrect response
[12]Mia: Oh. I thought it was nine.	
[13]Amilia: So what's it gonna be?	
[14]Mia: It's gonna be fifteen.	
[15]Amilia: Fifteen? (Puts 5 red counters on the empty ten frame)	[15]Querying an incorrect response
[16]Mia: No. Thirteen.	
[17]Amilia: There is another way you reckon? Thirteen. (Places counters on the empty ten frame) Let's have a look. You've got eight.	[17]Querying a correct response
[18]Mia: Let's take up two.	
[19]Amilia: Two up. (Moves the two red counters from the empty ten frame to the 8-dot ten frame to make 10)	
[20]Mia: That's thirteen.	
[21]Amilia: Thirteen. Good girl.	[21]Confirming, highlighting and privileging a correct response [21]Affirming

In turns 1, 3 and 5, the teacher, Amilia, presents a task. After presenting the task, Amilia provides wait-time (*post-posing wait-time)* of 12 seconds duration. In turn 6, the student, Mia, gives an answer incorrectly by writing 14 in the box. Amilia, in turn 7, first gives a 'neutral feedback' by saying, "Okay", then queries the way Mia solves the task. More queries occur in turns 11 and 15. In these turns the teacher queries Mia's answers in an attempt to encourage active learning. Thus, instead of being immediately corrected, the student eventually can realise her mistake and thereby solve the task. In this way, the teacher seems to be a *discourse* *prosthesis* (Graesser et al., 1999, p. 36) who endeavours to get the student to explain her understanding and the strategies she used to solve the problem. Mia flounders throughout turns 8, 10, 12 and 14 in attempting to solve the task. Amilia supports Mia by unscreening the ten frames in turn 11 (using the Key Element of *directing to check*) and by supplying cues and clues, and, in turn 15, by placing the five counters on the empty ten frame (using the Key Element of *querying an incorrect response*. These supports lead to the successful solution to the task. In turn 14, Mia seems to lose track of the task when she answers fifteen, so the teacher uses red counters to illustrate the task of 8+5 on the ten frames. In turn 16, Mia builds on this suggestion. At this point, she solves the task. In turn 17, Amilia questions Mia about her answer in order to determine Mia's solution method and to gauge Mia's certitude by asking for another way to solve the task (using the Key Element of *querying a correct response*). Amilia gives immediate feedback (using the Key Element of *confirming, highlighting and privileging a correct response*) in turn 21 after Mia completes solving the task. The feedback not only confirmed the correctness of Mia's answer, but also give her motivation in learning.

7.2.3.2 Modelling-Scaffolding-Fading

In the modelling-scaffolding-fading teaching strategy, the teacher usually demonstrates to the student how to solve a task, watches as the student practices portions of the task, and then provides necessary support until the student is able to accomplish the task independently (Collins et al., 1991). This model corresponds to a cluster of Key Elements including *directly demonstrating*, *scaffolding during* – sometimes with other support such us *querying an incorrect response*, *re-posing the task*, *rephrasing the task* and *directing to check* – and *recapitulating* and *explaining*. The model is evident in the data of the current study, but with a relatively low frequency. This might be because of the low frequency of the Key Element of *directly demonstrating*. The low frequency of this Key Element might reflect the teachers' desire for the student to engage in active, rather than passive, learning.

Scenario 7.2 (see Figure 7.2) describes a short tutorial dialogue between the teacher, Ava, and the student, Ella focusing on five-plus tasks using a combination of five-plus facts, which are presented with five-wise ten frames and arithmetic rack settings. The scenario is followed by a discussion of the relation between the use of the Key Elements referred to above and the teaching strategy of modelling-scaffolding-fading.

Figure 7.2 Scenario 7.2 Ava-Ella

Scenario	Key Elements
[1]Ava: Okay. So these are all our five plus facts. (Picks up all the five-plus facts - ten frame cards from the table). What I'm gonna do is use some of these (refers to the five-plus facts). You have to show it to me on there (refers to the arithmetic rack) and you have to tell me the answer. Alright? So I'll do the first one. (Places out a five-plus-two ten frame card on the table). Five (slides five red beads on the upper row from right to left) and two (slides two blue beads on the upper row from right to left) is seven. Five and two is?	[1,3]Directly demonstrating
[2]Ella: Seven.	
[3]Ava: Do you get the idea? (Slides all the seven beads on the upper row back to the right)	
[4]Ella: (Nods)	
[5]Ava: Alright. (Places another five-plus-five ten frame on the table) Five and five?	
[6]Ella: (Looks at the five-plus-five ten frame and then slides five red beads from right to left)	
[7]Ava: (Looks at Ella)	
[8]Ella: (Slides the five red beads from the left to the middle of the upper row of the rack, then slides them back to the left, and then looks at the five-plus-five ten frame again)	
[9]Ava: So say this one. (Points at the five-plus fact).	[9,11]Scaffolding during
[10]Ella: (Slides five blue beads on the upper row from right to left)	
[11]Ava: Now say it.	
[12]Ella: Five and five is ten. (Looks at Ava and smiles)	
[13]Ava: Good. (Places next five-plus fact on table)	
[14]Ella: (Looks at the fact for a couple of seconds and slides nine beads on the upper row from right to left in one push). Five and four is nine.	
[15]Ava: You know that one's really easy, don't you? (Points at the nine beads)	[15,17,19,21,23]Querying a correct response
[16]Ella: (Nods)	
[17]Ava: There's one empty here (points at the fact), if it was filled it would be…	
[18]Ella: Ten.	
[19]Ava: Ten. But there's one empty. And if they were all across (slides the last blue bead on the upper row from left to right) it would be…	
[20]Ella: Ten.	
[21]Ava: Ten. But there's one over here (slides the last blue bead back to the left), so it must be?	
[22]Ella: Nine.	
[23]Ava: Nine. Alright. Good girl.	

In turns 1 and 3, the teacher, Ava, models a method for solving an initial task of a sequence of tasks (using the Key Element of *directly demonstrating*). In turns 6, 8, 10 and 12, Ella attempts to solve the task, five and five, on the rack. In turns 9 and 11, Ava provides support to Ella by asking her to read aloud the five-plus fact (using the Key Element of *scaffolding during*). In the next task, five and four on the rack, in turn 14, Ella solves the task quickly and her strategy is spontaneous without assistance from Ava. In turns 15, 17, 19, 21 and 23, Ava queries Ella by asking questions in order to gauge Ella's strategy and understanding.

7.2.3.3 Sophisticated Motivational Techniques

Sophisticated motivational techniques include positive feedback, humour and motivation and are evident in the teaching sessions across the teachers. These techniques are reflected in the use of the Key Elements of *affirming, confirming, highlighting and privileging a correct response* and *giving encouragement after a partly or nearly correct response*. Some typical statements when using these techniques in the present investigation are listed below.

Positive feedback: "Well done! You're clever!"; "Ohh… (Teacher and student high-five) Amazing! Amazing!"; "Excellent! Fantastic!"; "Ohh… Can't trick you."

Humorous statements: "Ohh… (High-five) You had maths juice for breakfast?"

Motivational statements: "Oh, my goodness me. Let's have a look. Absolutely perfect. Every single one of them. (Ticks every sum on page)"

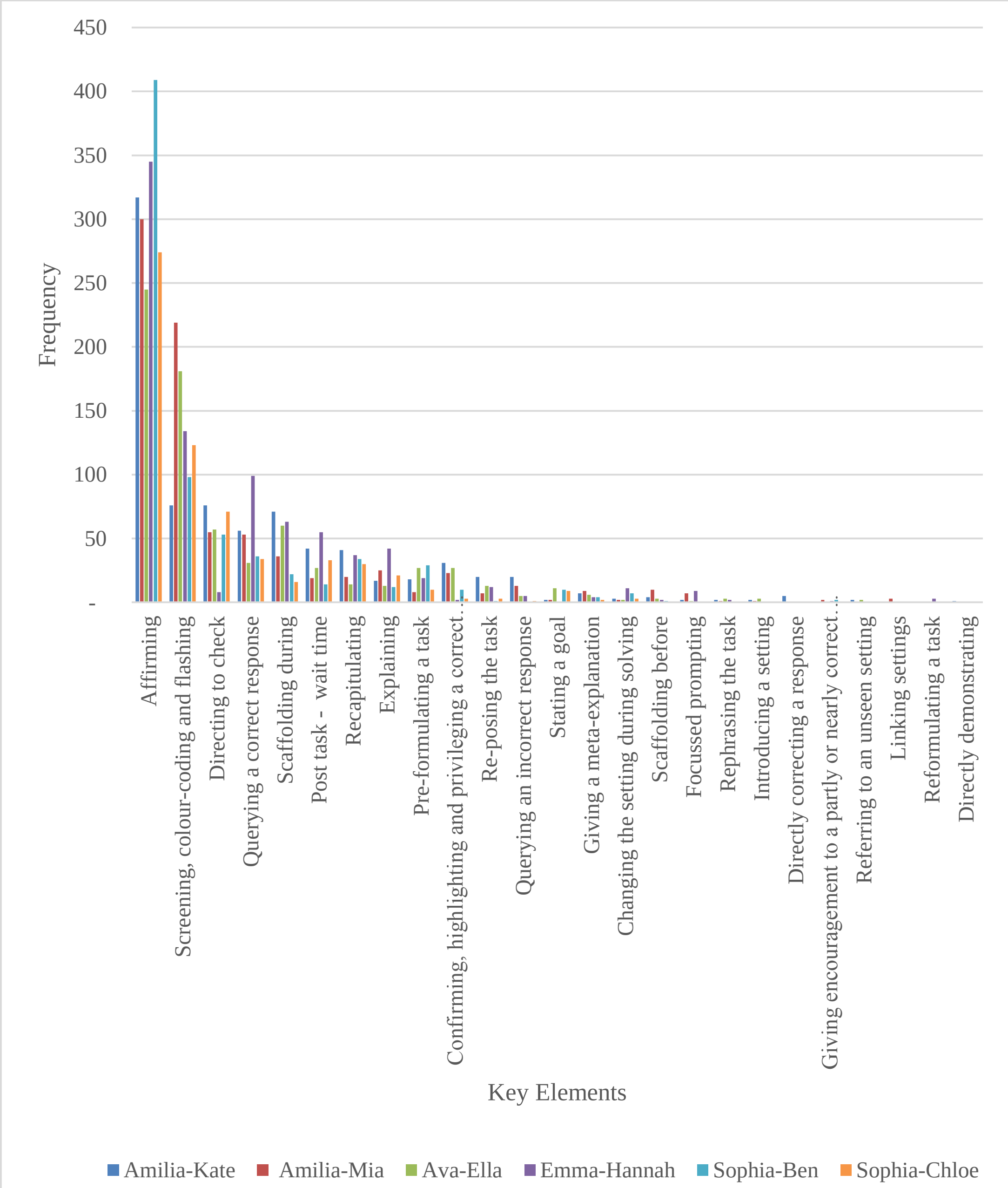
"Good. Let's have a look. (Ticks all the sums on page). Excellent! Excellent! Excellent! Excellent! (Finds a sticker and places on page). You need to have an excellent sticker. Good girl!"

7.3 Discussion of the Key Elements in Relation to Their Frequency

7.3.1 Frequency of the Key Elements

The frequency of each Key Element across the teaching sessions is a factor in determining the distribution and prevalence of Key Elements, regardless of the length and the amount of time spent when using the Key Elements. Table 4.4 (Chapter 4) shows the frequency of using Key Elements across the participating teachers. These are arranged from the largest to the smallest in terms of the total use. The frequency of using Key Elements across the participating teachers are represented graphically in Figure 7.3.

Figure 7.3 Comparison of Key Elements used across the participating teachers



In Table 7.2, the Key Elements are arranged in four groups according to how frequently they are used by the participating teachers (information derived from Table 4.4). Group 1 involves the Key Elements that at least one of the four teachers used at least 100 times. Group 2 involves the Key Elements that at least one of the four teachers used from 50 to 99 times. Group 3 involves the Key Elements that at least one of the four teachers used from 20 to 49 times and Group 4 involves the remaining Key Elements used less than 20 times.

Table 7.2 Key Elements grouped according to frequency

Group 1 ($x \geq 100$)	Group 2 ($50 \leq x < 100$)	Group 3 ($20 \leq x < 50$)	Group 4 ($x < 20$)
Affirming	Directing to check	Recapitulating	Giving a meta-explanation
Screening, colour-cording and flashing	Scaffolding during	Confirming, highlighting and privileging a correct response	Scaffolding before
	Querying a correct response	Querying an incorrect response	Changing the setting during solving a task
	Post-task wait-time	Re-posing the task	Focussed prompting
		Pre-formulating a task	Introducing a setting
		Explaining	Rephrasing the task
			Stating a goal
			Giving encouragement to a partly or nearly correct response
			Reformulating a task
			Referring to an unseen setting
			Linking settings
			Directly demonstrating
			Directly correcting a response

The frequency of each Key Element is considered to be a factor in determining the significance of the Key Element, but it does not necessarily follow that the more frequently a Key Element is used, the more effective it is in terms of learning outcomes. This claim is consistent with that of Ohlsson et al. (2007) that the frequencies of tutoring moves do not necessarily reveal their causal efficacy. Although the current study does not focus on determining the effectiveness of the Key Elements on student learning, the results presented here are not at odds with the findings of Ohlsson et al. (2007), in the sense that the most frequently used Key Elements are not necessarily regarded as the most important ones for teachers to use in all situations.

The Key Elements in Group 1, for example, are the most frequently used but this does not mean that they are the most useful Key Elements in terms of student learning. In a similar vein, the Key Elements in Group 4 are least frequently used but this does not mean that they are the least useful in terms of student learning. Differences in relative frequencies of the Key Elements might result from the extent to which the Key Element is relevant to a range of instructional situations. For example, the most frequently used Key Element, *affirming*, can be used in all tasks during working towards the solution or after the student has solved the task, whereas, other Key Elements can only be used in particular instructional situations.

The relatively low frequency of *directly correcting a response* and *directly demonstrating* may reflect a preference on the part of teachers to engage the student in active rather than passive learning. As well, a preference on the part of teachers for active learning could explain the high frequency of *scaffolding during*, *querying a correct response* and *querying an incorrect response*.

In the present investigation, the frequency of Key Elements is used to consider the extent to which different Key Elements are prevalent for different teachers. That is, the extent to which some Key Elements are used more frequently by some teachers and not others. As well, the frequency of Key Elements is used to consider whether some teachers have an ability to use a wider range of Key Elements than others, given a similar instructional situation. In Table 7.3, the teachers are categorised into *high user*, *moderate user* and *low user* of Key Elements. A *high user* is a teacher who uses a wide range of Key Elements frequently. A *moderate user* is a teacher who uses a moderately wide range of Key Elements and a *low user* is a teacher who uses a narrow range of Key Elements infrequently.

Table 7.3 The usage categories for each teacher-student dyad

High user	Moderated user	Low user
Amilia – Kate	Amilia – Mia	Sophia – Ben
Emma-Hannah	Ava-Ella	Sophia - Chloe

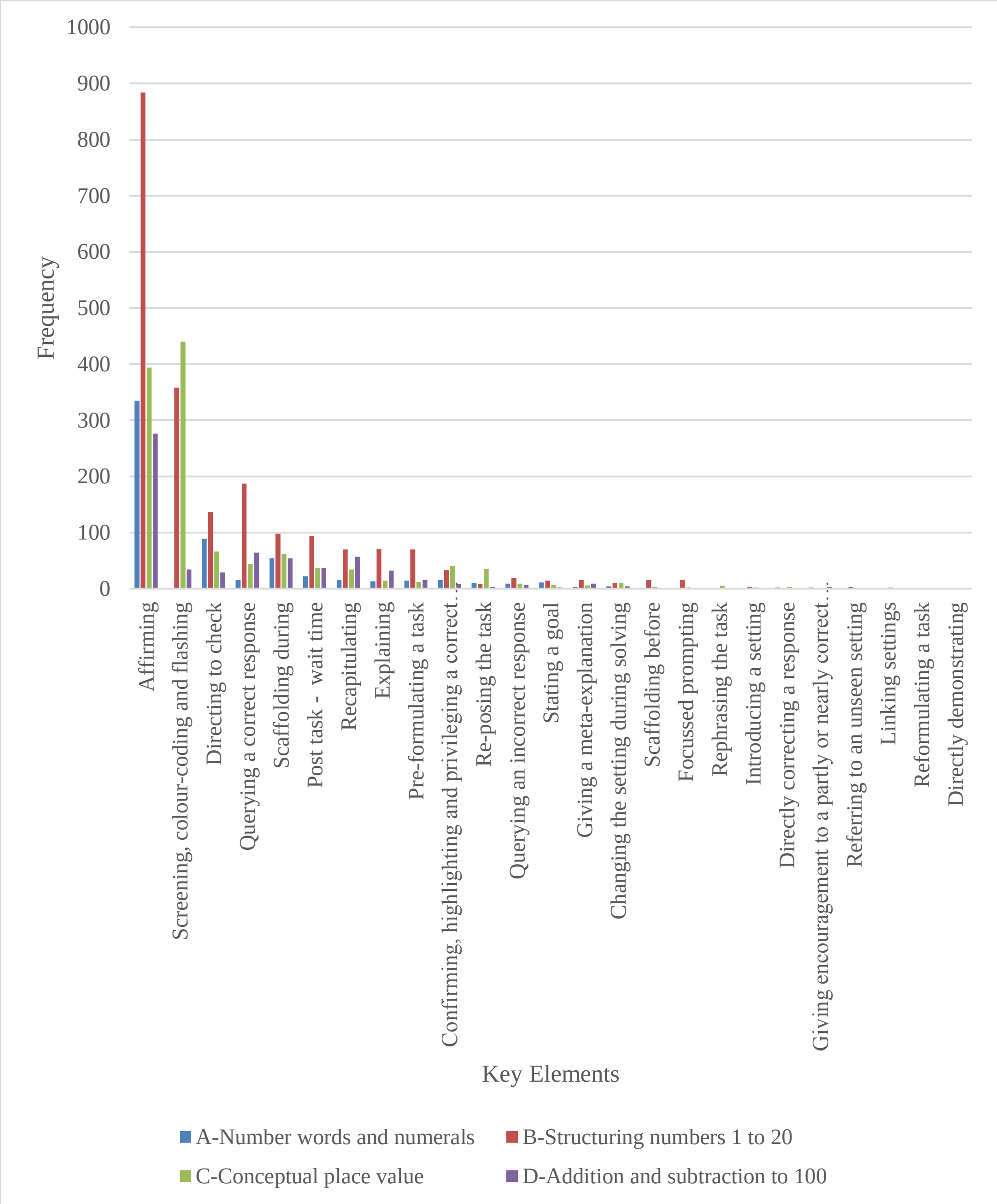
Table 7.4 below shows the frequencies of the Key Elements used across the four learning domains: A–number words and numerals; B–structuring numbers 1 to 20; C–conceptual place value; and, D–addition and subtraction to 100.

Table 7.4 Key Elements used across the learning domains

Key Elements \ Learning Domains	A- number words and numerals	B- structuring numbers 1 to 20	C- conceptual place value	D- addition and subtraction to 100
Affirming	335	884	394	276
Screening, colour-cording and flashing	0	358	440	34
Directing to check	89	136	66	29
Querying a correct response	15	187	44	64
Scaffolding during	54	98	62	54
Post task - wait time	22	94	37	37
Recapitulating	15	70	34	57
Explaining	13	71	14	32
Pre-formulating a task	14	70	12	16
Confirming, highlighting and privileging a correct response	15	33	40	8
Re-posing the task	10	8	35	3
Querying an incorrect response	9	19	9	7
Stating a goal	11	14	7	2
Giving a meta-explanation	3	15	6	9
Changing the setting during solving	4	10	10	4
Scaffolding before	1	15	3	1
Focussed prompting	1	16	2	0
Rephrasing the task	1	1	5	1
Introducing a setting	1	3	2	0
Directly correcting a response	2	0	3	0
Giving encouragement to a partly or nearly correct response	2	0	0	3
Referring to an unseen setting	0	3	1	0
Linking settings	0	1	2	0
Reformulating a task	0	1	1	1
Directly demonstrating	0	1	1	0

The Key Elements used across the dyads are represented graphically in Figure 7.4.

Figure 7.4 Key Elements used across the learning domains



7.3.2 Underused Key Elements

From the researcher's perspective, some Key Elements identified and described in this investigation are considered as high quality forms of practice in intensive, one-to-one instruction. Some Key Elements, however, were used effectively by some participating teachers but not by others. These are called underused Key Elements. According to the frequencies of Key Elements used, as shown in Table 4.4 (Chapter 4), Key Elements considered to be underused are *querying an incorrect response*, *changing the setting during solving*, *stating a goal*, *giving encouragement to a partly or nearly correct response*, and *focused prompting*.

Table 7.5, using information extracted from Table 4.4 (Chapter 4), shows the frequency of Key Elements which were regarded as underused.

Table 7.5 Frequency of underused Key Elements across the dyads

Key elements \ Teachers	Amilia-Kate	Amilia-Mia	Ava-Ella	Emma-Hannah	Sophia-Ben	Sophia-Chloe
Querying an incorrect response	20	13	5	5	0	1
Re-posing the task	20	7	13	12	1	3
Changing the setting during solving task	3	2	2	11	7	3
Focussed prompting	2	7	1	9	0	0
Introducing a setting	2	1	3	0	0	0
Rephrasing the task	2	1	3	2	0	0
Stating a goal	2	2	11	0	10	9
Giving encouragement to a partly or nearly correct response	0	2	0	1	2	0

An example of an underused Key Element is *querying an incorrect response*. According to Table 7.5, this Key Element was used quite often by Amilia with Kate (20 times) and often with Mia (9 times); sometimes by Ava with Ella (5 times) and Emma with Hannah (5 times); and almost no times by Sophia with both students Ben (none) and Chloe (one time).

A question that arises in the analysis phase is: Why does the use of Key Elements differ significantly among teachers? In other words, why does one teacher use particular Key Elements and another does not? This could be related to whether a teacher was not aware of the existence of a Key Element, or because that teacher did not have enough practice with using those Key Elements. These questions all relate to the particular expertise required to use these Key Elements. With this in mind, Section 7.3.3 describes the expertise entailed in selecting and using particular Key Elements. The findings of the study suggest that, if teachers are provided with appropriate professional learning and are given sufficient time, they can learn to select and use Key Elements which require particular expertise.

7.3.3 Grouping the Key Elements in Relation to Their 'Routineness'

The Key Elements can be arranged into two groups according to whether or not they are routinely used.

7.3.3.1 Key Elements Used in a Routine Manner

What seems to be the case is that teachers with significant levels of mathematical content knowledge, pedagogical content knowledge and teaching experience use particular Key Elements spontaneously, perhaps with little awareness that they are using them. Examples of such Key Elements are *affirming* and *confirming, highlighting and privileging a correct response*. In this case, the Key Elements are used by the teachers in a routine manner.

7.3.3.2 Key Elements Used in a Non-routine manner.

The successful use of each Key Element involves how, when and why the Key Element is used, and it seems that, for some Key Elements, teachers require particular expertise in order to be able to use them appropriately. In the present investigation, such Key Elements are not routinely used. Here these are called non-routine Key Elements. The expertise involves teacher professional noticing and dimensions of mathematisation, and, in this investigation, both are used by the teachers to unpack the in-the-moment decision making associated with using the Key Elements. This is not to say that professional noticing and dimensions of mathematisation are the only expertise required in the use of Key Elements, but the expertise related to these two areas was a central feature of the examination of Key Elements and their use in this investigation. The observations made in this investigation indicate that these two areas of expertise are needed in order for teachers to use effectively a particular Key Element in a specific instructional situation.

The study hypothesises that teachers with more expertise in professional noticing and the dimensions of mathematisation will be better able to use a wide range of Key Elements appropriately. Questions that arise are: Which Key Elements almost always require professional noticing? And, Which Key Elements almost always require a dimension of mathematisation for their effective use? This is discussed in detail in the following sections.

As described in Section 2.1.2.3 (Chapter 2), Jacobs et al. (2010, p. 169) defined teacher professional noticing of students' mathematical thinking as three interrelated skills. These skills are described by Jacobs et al. (2010, pp. 172–173) as follows. Attending to students' strategies (A) refers to the extent to which teachers attend to a particular aspect of instructional situations such as the mathematical details in students' strategies. Interpreting students' understanding (I) refers to the extent to which teachers interpret students' understanding as reflected in their strategies. Deciding how to respond (D) refers to the reasoning that is used by the teacher when deciding how to respond on the basis of students' understanding. Accordingly, later in the present investigation, the use of these interrelated skills is called the AID process.

Table 7.6 shows a rearrangement of the Key Elements according to the routineness of use of the Key Elements. Also, the non-routine Key Elements are rearranged into two groups: the Key Elements that require a low AID process and those that require a high AID process.

Table 7.6 Key Elements grouped according to their routineness

Key Elements		
Routine	**Non-routine**	
	Require low AID process	**Require high AID process**
Affirming	Screening, colour-coding and flashing	Scaffolding during
Confirming, highlighting and privileging a correct response	Directing to check	Querying an incorrect response
	Post task - wait time	Changing the setting during solving a task
	Introducing a setting	Giving encouragement to a partly or nearly correct response
	Scaffolding before	Rephrasing the task
	Reformulating a task	Re-posing the task
	Recapitulating	Focussed prompting
	Explaining	Querying a correct response

7.3.3.3 Linking Key Elements and Professional Noticing

This section focuses on the extent to which the teacher requires expertise in professional noticing in order to use effectively a particular Key Element in a specific instructional situation. Thus, in a sense, it is not about an analysis of what teachers notice, but rather what the author as a researcher notices when studying Key Elements in relation to teacher professional noticing.

First, when using any Key Element, the teacher requires the skill of attending to a student's strategies. This allows the teacher to capture observable, noteworthy aspects of the student's mathematical strategies. The information that the teacher perceives through attending to the student's strategies is helpful in the next two stages: interpreting the student's understanding and deciding how to respond to the student on the basis of their understanding. Based on the result of the investigation's research on the framework of Key Elements, particularly in the stage C–During solving a task, the information that the teacher can gain from the attending stage was categorised as: (a) student engagement in solving a challenging task; and (b) the student answering incorrectly. In the following section the two categories are explained in detail.

1. **Student engagement in solving a challenging task.** At this stage, the skill of interpreting the student's understanding is essential. This skill allows the teacher to interpret what they have observed from the attending stage in order to understand what is preventing the student from solving the problem. Below, four possible subsequent teacher responses are described.

   (i) The student is challenged in solving the task because they have lost track of some details of the task. The teacher's response could include using the Key Element of *re-posing the task* to help the student fully understand the task or to remind the student of some details of the task.

   (ii) The student is challenged in solving the task because they do not understand the task clearly in terms of its mathematical aspects or verbal expression. The teacher's response could include using the Key Element of *reformulating the task* or *re-phrasing the task*. Thus, the teacher expresses the task differently in order to make the meaning clearer for the student without changing the task.

   (iii) The student commences solving the task and indicates a desire to proceed with the task, but still has difficulty figuring out an appropriate method to solve the task. The teacher's initial response could be to use the Key Element of *post-posing wait-time* or *post-responding wait-time* rather than giving the student any support. If providing wait-time is not successful, the Key Element of *scaffolding during* or *focused prompting* could be used.

   (iv) The student apparently reaches an impasse, that is, the student is unable to solve the task that they are currently attempting. Depending on the specific circumstances, the teacher's response could involve using one or several of the following Key Elements in turn: *scaffolding during*, *focused prompting*, *re-posing the task*, *re-phrasing the task* or *changing the setting during solving*.

2. **The student answering incorrectly.** Similar to the situation where the student is engaged in solving a challenging task, at this stage, the skill of interpreting the student's understanding is essential. This skill allows the teacher to interpret what they have

observed in terms of the student's strategy, in order to ascertain reasons for the incorrect answer and to evaluate the answer. Below two possible subsequent teacher responses are outlined.

(i) The student made an error in one or more steps. The teacher's response could involve using the Key Element of *querying an incorrect response*, that is, the teacher questions the student about their answer with the purpose of helping the student realise their error and solve the problem. In some cases, the teacher's response could involve using *directing to check* with the purpose of indirectly assisting the student to solve a task.

(ii) The student gave an incomplete answer, but from the teacher's perspective the student is on the right track and they might be able to solve the task with reasonable support. The teacher's response could involve using the Key Element of *giving encouragement to a partly or nearly correct response*. This could involve indicating that the student is on track, confirming the correct part, and then providing scaffolding. Concurrently, the teacher would encourage the student to continue without being overly concerned about the student's inadequate response. This response typically has the purpose of keeping students on track and giving them more motivation and confidence to continue solving the task.

In task-solving situations, students' signals are often tacit. These can challenge the teacher in interpreting the student's understanding, in order to make an appropriate decision about how to respond. Teachers, therefore, might fail in their attempts to support the student by using particular Key Elements, because the teacher does not understand the student. Such support might interfere with the student's thinking. Five of the problematic teacher behaviours described earlier including unnecessarily *reformulating a task, interrupting the student, inappropriately re-posing, rushing or indecent haste* and *miscuing* are cases of unsuccessful use of Key Elements in attempting to support the student. Therefore, the expertise developed in relation to professional noticing is essential for teachers to use the Key Elements effectively.

7.3.3.4 Linking Key Elements and Dimensions of Mathematisation

As described in Section 2.1.2.4 (Chapter 2), Ellemor-Collins and Wright (2011b) described ten dimensions of mathematisation in intensive, one-to-one instruction. This section focuses on discussion of the linking of Key Elements and a dimension of mathematisation—*distancing the setting*. In the case of *distancing the setting*, for example, a teacher can progressively distance the setting by instructing a student through steps, such as: (i) manipulating the physical materials; (ii) seeing the materials but not manipulating them; (iii) seeing them only momentarily; and (iv) solving tasks posed in verbal or written form without materials. This dimension is reflected in the use of some Key Elements when the teacher provides support by changing the setting during solving the task. The following section provides an extended discussion focusing on the link between the Key Element of changing the setting during solving the task to professional noticing and a dimension of mathematisation – *distancing the setting*.

7.3.3.5 An Extended Discussion of Linking One Key element, Changing the Setting During Solving, to Professional Noticing and Dimensions of Mathematisation

The Key Element of *changing the setting during solving* seems usually to require a high level of expertise in professional noticing and dimensions of mathematisation—*distancing the setting*. Thus, this section focuses on unpacking the in-the-moment decision making associated with using *changing the setting during solving* in the light of professional noticing and dimensions of mathematisation, particularly *distancing the setting*.

As in its description, *changing the setting during solving* is intended to be used when the student apparently reaches an impasse. In order to perceive that the student apparently reaches an impasse, the teacher needs to attend to the student's strategies carefully. The teacher then needs to interpret the student's understanding in order to determine how to change the setting. In changing the setting the teacher would intentionally introduce new elements which, from the teacher's perspective, can be linked to elements in the original setting (Wright et al., 2002). Thus the intention on the teacher's part is that the new elements enable the student to reconceptualise the current task, and arrive at a solution which was not available to the student before the change in the setting. In some cases, the teacher changes the setting to no avail. This often occurs because the student is not able to conceive of the links between the current and previous settings, although these links might be very evident to the teacher. Thus, interpreting appropriately the student's thinking and answer is a significant step for the teacher to determine the change to be made to the setting.

The expertise of using a dimension of mathematisation, *distancing the setting*, is taken into account at the stage of changing the setting, although in using the Key Element of changing the setting during solving, the teacher would instruct the student through the steps in reverse. For instance, the teacher initially poses the task verbally or in written form and the teacher perceives that the student seems to have reached an impasse. The teacher might display the relevant setting momentarily or allow the student to manipulate the materials.

7.3.3.6 Illustration of the Links Among Professional Noticing, Dimensions of Mathematisation and the Use of Key Elements

In this Section, Scenario 7.3 (see Figure 7.5) is used to illustrate how professional noticing and *distancing the setting* is used to unpack the in-the-moment decision making associated with using the Key Elements, particularly *changing the setting during solving*. This Scenario is the same as Scenario 5.2 (Figure 5.10). It is reused in this section because it contains a rich diversity of Key Elements and illustrated well for the case.

Scenario 7.3 (see Figure 7.5) focused on decrementing by 100s and involved the teacher, Sophia, and her student, Ben. Sophia initially posed a task verbally; subsequently, she used the setting of arrow cards and then changed to the setting of dot materials. The scenario is followed by a discussion on how professional noticing is used to unpack the in-the-moment decision making associated with using the Key Elements.

Figure 7.5 Scenario 7.3 Sophia – Ben

Scenario	Key Elements
Sophia: What's a hundred less than a thousand and fifty? Ben: .... (After 10 seconds) One hundred and fifty.	Post-posing wait-time
Sophia: (Looks at Ben) Ben: No... (After 16 seconds) What did you say again?	Post-responding wait-time
Sophia: One thousand one hundred and fifty. Then a hundred less. Ben: (After 9 seconds). Three hundred and fifty? No. Ninety fi-, ninety f-, one hundred and, no, nine hundred and five. No. One hundred and five.	Re-posing the task
Sophia: (Looks at Ben and smiles encouragingly) Nearly, I think you've. Nearly there.	Giving encouragement to a partly or nearly correct response
Ben: What did you say it was...?	[Ben appeared to reach an impasse]
Sophia: So, it's one thousand. (Brings arrow card sheet in front). Can you make one thousand and fifty? See what it looks like. Ben: (Makes up the number)	Changing the setting during solving
Sophia: Now, a hundred less. Ben: … no hundreds in this Sophia: yes, so where could you take the hundred from?	Scaffolding during
Ben: Oh, the fifty? No. You take, you taking the hundred from a thousand? Sophia: Mm hmm. So how many is that? How many would I have left of that a thousand if I took a hundred away from it?	Scaffolding during
Ben: Fif-, no f-, five hundred. No. Sophia: Do you want to make it with the dots and see? Ben: Mmm. Sophia: Yep. (Gets plastics back out). One thousand and fifty, so you've got to make a thousand and fifty. Ben: (Lays out 100-squares on table) Sophia: Mm hmm. (Hands the ten-dot strips to Ben) Ben: (Lays out five 10-dot strips)	Changing the setting during solving
Sophia: Right, so how many have you got? How many dots? Ben: One thousand and fifty.	Scaffolding during
Sophia: Mm hmm. So you want a hundred less. Ben: (Takes one hundred-dot card away) Nine hundred and fifty.	Scaffolding during
Sophia: Good, Ben. Well done.	Affirming
Sophia: (Gets 1050 arrow card number). So, you had one thousand and fifty. Yeah? Ben: Mmm. Sophia: Where did you take the hundred from? Ben: From the one thousand. Sophia: Mm hmm. And when you took that one hundred away what did you have left? Ben: Nine thousand and fifty.	Recapitulating

Scenario	Key Elements
Sophia: Mmm… Ben: No, one hundred and fifty. Nine hundred and fifty. Sophia: Nine hundred and fifty. What does nine hundred look like? Ben: Umm Sophia: (Gets out arrow card sheet). There's nine hundred (points to it). Ben: (Takes 900 from sheet and starts to make up the number) Sophia: Then you ... Ben: Oh, fifty. (Grabs 50 arrow card).	
Sophia: That's it. Good on you. That's it. Well done. That's good.	Affirming

Sophia initially posed the task of "What's a hundred less than a thousand and fifty?" verbally and looked intently at Ben and waited for 10 seconds (*post-posing wait-time*). Ben answered incorrectly "One hundred and fifty". Rather than comment on Ben's answer, Sophia continued to look at Ben and waited for 16 seconds (*post-responding wait-time*). Ben then asked "What did you say again?" Sophia responded to Ben's request by *rephrasing the task* and waited for 9 seconds. Ben answered "Three hundred and fifty", but immediately changed his answer to "Ninety fi-, ninety f-, one hundred and, no, nine hundred and five". Thus, on two occasions, Ben immediately changed his answer. On both occasions, Sophia's response was to attempt to keep Ben on track by *giving encouragement to a partly or nearly correct response* – "Nearly, I think you're nearly there". Ben again asked Sophia to repeat the task. After all attempts to help Ben solve the task so far, Sophia perceived that Ben had apparently reached an impasse. Sophia decided to change the setting by bringing out a sheet of arrow cards and asked Ben to make the number 1050 using the arrow cards. Along with changing the setting, Sophia provided *scaffolding during* to help Ben solve the task. After eight seconds, during which Ben attempted to solve the task using arrow cards, it seems that Ben would not be able to solve the task. Sophia presented another setting–dot materials involving 100-squares and 10-strips. Sophia continued to scaffold to support Ben. The setting of dot materials seemed to support visualisation related to the number 1050 and this enabled Ben to reconceptualise the task by removing a 100-square. For a total of approximately four minutes, Ben was engaged in a sustained period of highly interactive, one-to-one instruction which culminated in him solving the task of 100 less than 1050. After the task was solved, Sophia briefly summarised the process of how the task was solved (*recapitulating*), as well as linking the two settings of arrow cards and dot materials (*linking settings*) in order to emphasise crucial features of the student's strategy and consolidate the student's learning.

The linking of *distancing the setting* and the use of the Key Element of *changing the setting during solving* is also illustrated in Scenario 5.2 where the teacher, Sophia, used the Key Element of *changing the setting during solving* twice. In doing so, the instruction progressed from the initial setting (verbally) to the second setting (notation represented by using arrow cards) to the last setting (dot materials). This process of mathematising can be seen as a reversal of distancing the setting. Initially the setting is verbal only, then the setting consists of the arrow cards and finally the setting consists of dot materials. Thus, rather than distance the setting, the setting is progressively 'un-distanced'.

7.4 Concluding Remarks

This chapter provided a comprehensive summary of the key findings of the investigation and explained how valuable the findings are and why they are significant. As well, the findings were discussed comprehensively in relation to the broader educational and mathematical research literature focusing on one-to-one instruction. A further discussion of the frequency of the Key Elements used by the participating teachers was provided. In addition, the link between the use of the Key Elements and the expertise including teacher professional noticing and dimensions of mathematisation were discussed.

Chapter 8 – **Conclusion**

This investigation was conducted in the context of the Mathematics Intervention Specialist Program (Wright et al., 2011), where teachers undertake specialist training focusing on intensive, one-to-one instruction with low-attaining Years 3 and 4 students. The investigation set out to identify and illuminate Key Elements that teachers used when interacting with their students during teaching sessions. As well, the investigation aimed to conceptualise a framework for analysing intensive, one-to-one instruction. For those purposes, two research questions were addressed:

1. What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?
2. How can Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

This chapter synthesises the empirical findings of the investigation with respect to the research questions, discusses theoretical and methodological contributions that the investigation makes to the field, and explores the implications of the investigation for mathematics intervention programs and, in particular, for one-to-one intervention in primary schools. As well, the chapter acknowledges the limitations of the investigation and provides some suggestions for future research.

8.1 Synthesis of the Empirical Findings

In this section, the empirical findings from the investigation are synthesised with respect to answering the research questions. A summary of the findings is presented with regard to the overarching purpose of the investigation.

8.1.1 The Identified Key Elements of One-to-one Instruction

With respect to Research Question 1, 25 Key Elements were identified. Twelve Key Elements related closely to teacher behaviours that were already described in the research literature and another 13 Key Elements emerged during the analysis phase of the investigation. The 25 Key Elements were presented as Set A and Set B in Chapter 5 and are presented in Table 8.1 below.

Table 8.1 The 25 Key Elements identified in the present investigation

Set A	Set B
Directing to check	Recapitulating
Affirming	Giving a meta-explanation
Changing the setting during solving	Confirming, highlighting and privileging a correct response
Post-task wait-time	Re-posing the task
Introducing a setting	Rephrasing the task
Pre-formulating a task	Stating a goal
Reformulating a task	Querying an incorrect response
Screening, colour-coding and flashing	Focussed prompting
Querying a correct response	Giving encouragement to a partly or nearly correct response
Explaining	Referring to an unseen setting
Scaffolding before	Linking settings
Scaffolding during	Directly demonstrating
	Directly correcting a response

The investigation provided insight into the essence (Van Manen, 1997, p. xiv) of Key Elements of intensive, one-to-one intervention focusing on whole-number arithmetic to Years 3 and 4 students. For each of the 25 Key Elements, a rich and deeply layered description was developed. The description of each Key Element drew on a corpus of video recordings of teaching sessions. The 25 Key Elements constituted a set of Key Elements likely to be useful for analysing one-to-one instruction.

An additional outcome of the present investigation is a set of ten problematic teacher behaviours associated with one-to-one instruction. These problematic teacher behaviours were identified during the data analysis phase of the present investigation. The behaviours were listed in Table 5.4, Section 5.3. The problematic teacher behaviours occured in instructional contexts where the teacher was presenting a task, providing support, giving an explanation or giving feedback. In this case, a teacher might unwittingly behave in a way regarded as problematic. Thus, the teacher might not be consciously aware of their problematic teaching behaviour when it occured. The behaviour might be a consequence of their regular teaching manner which in turn might be influenced by, for example, their teaching experiences, their mathematical content knowledge, their pedagogical content knowledge or the teaching environment. Table 8.2 below presents the problematic teacher behaviours in relation to the instructional contexts where they are occurred.

Table 8.2 The ten problematic teacher behaviours identified

Instructional context occurred	Problematic teacher behaviours
Presenting a task	Flagging a task as being difficult Flagging a task as being easy Simultaneously making more than one request
Providing support	Interrupting the student Inappropriately re-posing Rushing or indecent haste Miscuing Red-herring
Giving an explanation	Non sequitur
Giving feedback	Giving a ‘back-handed’ compliment

By identifying and illuminating problematic teacher behaviours, this investigation contributed to making teachers explicitly aware of behaviours regarded as problematic. As well, while the 25 identified Key Elements can be regarded as good teaching practices, the 10 identified problematic teacher behaviours can be regarded as teaching practices to be discouraged. The identification of the problematic teacher behaviours, therefore, complements the collection of Key Elements in helping teachers to refine their teaching practices. One could argue that, in their teaching practice, it is equally important for teachers to avoid using problematic behaviours as it is for them to use the Key Elements.

8.1.2 Conceptual Framework for Analysing One-to-one Instruction

With respect to Research Question 2, a framework of Key Elements for analysing one-to-one instruction was conceptualised. The framework provided a context which enabled an understanding of how teachers use specific clusters of Key Elements to achieve particular pedagogical goals.

The framework is layered into four stages relevant to the teacher dealing with an arithmetic task. Table 8.3 below describes the framework for analysing one-to-one instruction in the Mathematics Intervention Specialist Program. The excerpts from Mathematics Intervention Specialist Program teaching sessions were presented in Section 6.2 (Chapter 6) to illustrate how the framework could be used to analyse one-to-one instruction. Those excerpts were regarded as representative scenarios corresponding to the four stages of the framework.

Table 8.3 Conceptual framework for analysing one-to-one instruction

Stage	Stage names	Sub-stages	Typical Key Elements
A	Before posing a task		Introducing a setting; referring to an unseen setting; pre-formulating a task; scaffolding before; stating a goal; directly demonstrating
B	Posing a task		Screening, colour-coding and flashing; reformulating a task
C	During solving a task	C1–Responding to a correct response	Affirming; confirming, highlighting and privileging a correct response; querying a correct response
		C2–Responding to a partly correct response	Giving encouragement to a partly or nearly correct response; directly correcting a response; scaffolding during; post-posing wait-time; post-responding wait-time; directing to check; querying an incorrect response; rephrasing the task; re-posing the task; focused prompting; changing the setting during solving
		C3–Responding to an incorrect response	
		C4–Responding to an impasse	
D	After solving a task		Recapitulating; explaining; giving a meta-explanation; confirming, highlighting and privileging a correct response; affirming

The framework of Key Elements could serve as a guide to leaders in mathematical instruction in their analysis of one-to-one instruction. Further, the framework could inform teachers working with low-attaining students in their professional practice by providing useful information about how teachers and students interact in mathematical interventions, which in turn may illuminate how particular practices influence student learning outcomes.

8.2 Theoretical and Methodological Contributions and Implications

While the present investigation provided a comprehensive methodological account of how the Key Elements of one-to-one instruction were identified and illuminated, and how the Key Elements could be used for analysing intensive, one-to-one instruction, it has also contributed theoretically and practically to the research field of intensive one-to-one instruction in arithmetic. This section presents the contributions and implications of the empirical findings of the investigation and explains how these findings may impinge on current understanding.

As stated in Section 1.2 (Chapter 1), tutoring research has not yet converged on a common set of expert tutoring strategies. Terms used to refer to tutoring strategies that expert tutors used when interacting with students vary from study to study. The present investigation conceptualized a definition of phenomena called Key Elements of one-to-one instruction as described in Section 1.2 (Chapter 1). This definition is an appropriate response to the challenge to converge on a term that could be used to refer to expert tutoring strategies.

By identifying and illuminating the Key Elements of one-to-one instruction, the investigation advances the understanding of teacher-student interactions and teaching practice in intensive, one-to-one intervention. This is because understanding the Key Elements can lead to more effective ways to characterise the range of instructional strategies teachers use. Further, an understanding of teacher-student interactions and teaching practice in intensive, one-to-one intervention could inform teachers working with low-attaining students by describing how teachers and students interact in mathematical interventions, which in turn may illuminate how particular practices influence student learning outcomes. As well, an understanding of the Key Elements would allow for extension and refinement of the research relevant to intensive intervention in the learning of whole-number arithmetic.

The empirical findings of the investigation provide a better understanding of teaching methods that expert tutors use when interacting with students. Such understanding can contribute to the body of research focusing on the study of expert versus non-expert tutoring (e.g., Cade et al., 2008; Lu et al., 2007). These findings might help to explain why expert tutors are more effective than non-expert tutors. As well, the findings on what Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program and how the Key Elements are used to respond to the students' mathematical understanding could inform the design of an Intelligent Tutoring System. An Intelligent Tutoring System is a computer-based tutoring program that is designed to provide immediate and customised instruction or feedback to students, usually without intervention from a teacher. The investigation, therefore, would contribute to computationally modelling expert tutoring in the Intelligent Tutoring System which focuses on tutoring students in whole-number arithmetic.

The outcomes of the investigation shed light on why Key Elements of one-to-one teaching give rise to successful learning outcomes. Although the investigation does not focus on the effectiveness of the Key Elements used by teachers on student learning outcomes, the Key Elements and their comprehensive descriptions could contribute to strengthening the evidence in supporting future research which focuses on the evaluation of a specific numeracy intervention. For example, in the case of the Mathematics Intervention Specialist Program, a future study might use the empirical findings in relation to the Key Elements from the present investigation to examine the efficacy and effectiveness of the intervention program in terms of improvements in students' mathematical performance. Therefore, the present investigation could be an appropriate response to the need to provide more rigorous data to evaluate the efficacy and effectiveness of numeracy intervention programs described as a research problem in Section 1.2 (Chapter 1).

The framework of Key Elements constitutes an extension of the current body of theoretical knowledge about targeted one-to-one intensive intervention in whole-number arithmetic. The framework could serve as a guide to instructional leaders in mathematical instruction in their analysis of one-to-one instruction. As well, intervention teachers could use the framework to reflect on their instructional practice, and instructional leaders could use the framework in professional learning settings. This, in turn, can benefit schools and education systems because the resulting framework could be applied to strengthening classroom, as well as intervention, instruction.

The Key Elements, therefore, should be introduced intentionally to teachers. In particular, the collection of Key Elements might be used as a resource in professional development in the Mathematics Intervention Specialist Program. Perhaps in a future Mathematics Intervention Specialist Program professional learning situation, one focus might be to feature a case study of Key Elements used during a course of teaching sessions. The case study could be presented at professional development meetings. Through analysis and discussion of the use of the Key Elements, teachers could learn from their own teaching and from that of their colleagues. By doing so, the teachers would be more aware of all the Key Elements including how and when to use each one. In this way, teachers could focus on using Key Elements that they had not used before.

8.3 Limitations of the Investigation

The purpose of this section is to acknowledge the limitations of the investigation. Three aspects of each limitation are described. The first is to announce the limitation. The second is to describe the nature of the limitation and explain the decisions were made during the research process in order to cope with the limitation. The third is to suggest (if applicable) how the limitations could point to the need for future research.

8.3.1 Methodological Approaches to the Research Questions

The first limitation involves a consideration of the suitability of the methodological approaches to the research questions. As stated earlier, the main focus of the investigation was divided into two phases. The first was to identify and illuminate Key Elements of one-to-one instruction used by a teacher when interacting with a student in solving an arithmetical task. The second was to conceptualise a framework of Key Elements for analysing one-to-one instruction. The two phases corresponded with the two research questions below.

1. What Key Elements are used during intensive, one-to-one instruction in a mathematics intervention program?
2. How can Key Elements be used to analyse intensive, one-to-one instruction in whole-number arithmetic?

The methodology of phenomenology was used to address Research Question 1 in which Key Elements were viewed as the central phenomenon requiring identification and illumination. The neatness of fit with phenomenology as a methodology for the present investigation derives from its capacity to permit repeated observation, and examination of certain teacher and student behaviours. In addition, the basic phenomenological technique is to reduce individual experiences of the participating teachers to their behaviours that constituted Key Elements, which in turn led to developing descriptions of the universal essence of the Key Elements (Van Manen, 1997, p. xiv). Therefore, phenomenology was appropriate for addressing Research Question 1. However, it is acknowledged that, to some degree, the methodology of phenomenology was modified in order to address Research Question 2. Instead of using different approaches to each research question, the researcher decided to use phenomenology for both. This was because, from the researcher's perspective, while with respect to Research Question 1, the Key Elements were examined individually, in the case of Research Question 2, the identified Key Elements were seen in an instructional context – a task block. That enabled an understanding of how teachers used specific clusters of Key Elements to achieve particular pedagogical goals. Therefore, to some degree, it could be seen how the teachers used the Key Elements in response to a particular instructional situation in an instructional context of a task block as a phenomenon requiring exploration and illumination when approaching Research Question 2.

8.3.2 Data Collection Method

The second limitation relates to the method used to collect data. After completing the interpretation of the Key Elements and the problematic teacher behaviours, the researcher realised that it would help if the researcher could come back to the participating teachers for a conversation in relation to their use of the Key Elements and the problematic behaviours. In this regard, the researcher and participating teachers could review some extracted video files of their teaching. That would provide an opportunity for a discussion among the researcher and the participating teachers in relation to the researcher's interpretations and the teachers' perspective on their use of the Key Elements and the problematic behaviours. In that way, the investigation would enable a triangulation that facilitated validation of data. A suggestion for future research is that the participating teachers should be involved to some degree in order to enhance triangulation of the data. As van Manen (1997) suggests, interviewing the participants in phenomenological human science could serve a very specific purpose, that is an interview "may be used as a vehicle to develop a conversational relation with a partner (interviewee) about the meaning of an experience." (p. 66).

The method of data collection used in this investigation could not involve discussion with the participating teachers. This was because, as stated in Section 3.4.2 (Ch.3), the video files were provided by the researcher's doctoral supervisor. The video files involving the teaching sessions and pre- and post-assessments were made some years ago with the informed consent of the teachers and students' parents for subsequent analysis for the purposes of professional development and research. The investigation, therefore, was a retrospective study. As well, in a commitment to the Human Research Ethics Committee, the identities of the students and the teachers were not disclosed to the researcher, nor were their identities in any way relevant to the proposed investigation.

As described above, the researcher was not able to discuss, with participating teachers, the analysis of their teaching. In order to address this deficiency, an academic supervisor of this investigation, Wright (2003, 2008), the scholar who originally developed the Mathematics Recovery program and who has worked and trained hundreds of expert tutors over many years including in the Mathematics Intervention Specialist Program, acted as key auditor, reviewing formatively the process of data analysis.

8.4 Directions for Future Research

The investigation provided insight into the essence of Key Elements of intensive, one-to-one intervention focusing on whole-number arithmetic to Years 3 and 4 students. That involved developing comprehensive descriptions of: (a) the essence of Key Elements (Van Manen, 1997, p. xiv); (b) the meanings and significance of the Key Elements; and, (c) how the teachers behaved when implementing the Key Elements in the context of one-to-one intervention teaching. Further research could focus on six directions as follows.

First, further research on Key Elements of one-to-one instruction could focus on non-verbal communication, including facial and eye expression; gestures, postures and touches; the interlinking of verbal and non-verbal elements; and teacher observation. Such research would complement the set of Key Elements identified in the present investigation resulting in a more comprehensive set of tutoring strategies used by expert tutors when interacting with a student in an intensive, one-to-one context.

Second, regarding the potential links between the use of the Key Elements and the students' progress in the intervention program, further research could focus on determining the salience of the Key Elements in terms of their efficacy for supporting student learning. As well, further research could focus on the comparison of the use of Key Elements in cases where students make significant progress with cases where students do not make significant progress. This could be examined in cases where the students were taught by the same teacher and also in cases where the students were taught by different teachers.

Third, one possible line of inquiry that seems to be encompassed within Research Question 2 might be to analyse sequences of Key Elements to uncover patterns of successive elements. For example, might querying an incorrect response (or some other Key Elements) lead to a predictable sequence or sequences of subsequent Key Elements? If a sequence pattern is observed across teachers for one task, is it stable across different tasks?

Fourth, if the researcher could play an active role in the Mathematics Intervention Specialist Program, an investigation could focus on evaluating the effectiveness of the Key Elements on student learning outcomes. In this case, the teachers could be divided into two groups: participants and counterparts. The participants would undertake special training in the use of the Key Elements and also would be introduced to the problematic behaviours, whereas, the counterparts would not. The comparison of students' progress between the two groups corresponding to the participant teachers and counterpart teachers might shed light on the effectiveness of the Key Elements on student learning achievement.

Fifth, in Section 7.3, the present investigation provided a discussion of the Key Elements in relation to their frequencies across the participating teachers. The discussion focused on the extent to which different Key Elements were prevalent for different teachers, that is, some Key Elements occurred more frequently for some teachers than for others. Further research could be conducted to examine to what extent particular teachers could be characterised in terms of their use of the Key Elements, that is, to what extent different teaching styles could be determined. Further research could also examine more deeply how some Key Elements are used more in particular learning domains.

Finally, the researcher comes from Vietnam where tutoring, particularly private tutoring, has become widespread throughout the country with "a current enrolment of more than 30 percent and 50 percent of primary and secondary students respectively" (Dang, 2011, p. 27). However, the tutors are normally unprofessional tutors such as peer tutors, school teachers or retired teachers. In the Vietnamese educational system, there are no programs that could provide some sort of professional training for someone who wants to be a professional tutor. As well, Vietnam lacks intervention programs that help children who are having difficulties in learning mathematics in particular and more generally in other school subjects. Therefore, the researcher wishes to undertake research in a cross-cultural context, focusing on the implications of the findings of the present investigation. As well, a professional development program, for example, the Mathematics Intervention Specialist Program, could be implemented in other contexts for training teachers to be specialist teachers in order to help the students with learning difficulties in mathematics.

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Appendices

Appendix 1 Australian Numeracy Interventions

Australian Numeracy Interventions		
Tier 1	**Origin**	**Source**
Count Me in Too (CMIT)	NSW	Bobis and Gould (2000); Stewart, Wright and Gould, (1998)
Count Me in Too Indigenous (CMITT)	NSW	Adapted the CMIT for Aboriginal children (Howard & Perry, 2002)
First Steps in Mathematics	Western Australia	Western Australian Minister for Education (2013)
Learning in Early Numeracy (LIEN)	NSW	Anderson (2006)
Mathematics in Indigenous Contexts Project	NSW	Howard, Perry, Lowe, Ziems, and McKnight (2003)
Numeracy Matters	NSW	http://www.dec.nsw.gov.au/about-us/news-at-det/news/mathematics-matters
Success in Numeracy Education (SINE)	Victoria	Clarke, Lewis, Stephen, and Downton (2005)
Taking off with Numeracy (TOWN)	NSW	Gould (2010)
Tier 2	**Origin**	**Source**
Best Start Targeted Early Numeracy (TEN)	NSW	http://www.curriculumsupport.education.nsw.gov.au/beststart/ten/general.htm
Extending Mathematical Understanding Intervention (EMU)	Victoria	Gervasoni (2002)

Getting ready in Numeracy (GRIN)	Victoria	Sullivan (2011)
Mathematics Intervention	Victoria	Pearn and Merrifield (1996)
Mathematics Recovery (MR)	NSW	Wright (2003, 2008)
Numeracy Intervention Project (NIP)	NSW	Thornton, Quinane, Galluzzo, and Taylor (2010)
Numeracy Intervention Research Project (NIRP)	Victoria	Ellemor-Collins & Wright (2009, 2011); Wright et al. (2007)
QuickSmart Numeracy	NSW	Pegg and Graham (2007)
Taking off with Numeracy (TOWN)	NSW	Gould (2010)
Train a Math Tutor Program	Queensland	Baturo and Cooper (2006)

Source: Meiers, Reid, McKenzie, & Mellor, 2013, p. 67.

Note: TOWN is both a Tier 1 and Tier 2 numeracy intervention program.

Appendix 2 Ethics Approval

Please note that the Chair of Higher Degrees Research Committee, under delegated authority, has approved for a change to the current research title: Targeted, One-to-one Instruction in Whole-number Arithmetic: A Framework of Key Elements.

Southern Cross University

HUMAN RESEARCH ETHICS COMMITTEE (HREC)
HUMAN RESEARCH ETHICS SUB-COMMITTEE (HRESC)

NOTIFICATION

To:	Professor Martin Hayden/Professor Bob Wright/Thi Le Tran School of Education t.tran.30@student.scu.edu.au,martin.hayden@scu.edu.au
From:	Secretary, Human Research Ethics Committee Division of Research, R. Block
Date:	29 June 2012
Project:	*Teachers' Professional Development through a Mathematics Intervention Specialist Program.*
	Approval Number ECN-12-163

The Southern Cross University Human Research Ethics Committee has established, in accordance with the National Statement on Ethical Conduct in Human Research – Section 5/Processes of Research Governance and Ethical Review, a procedure for expedited review and ratification by a delegated authority of the HREC.

This expedited application was considered by the Chair, HREC and has been approved.

All ethics approvals are subject to standard conditions of approval. These should be noted by researchers as there is compliance and monitoring advice included in these conditions.

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HREC Administration
Ph: (02) 6626 9139
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Professor Bill Boyd
Chair, HREC
Ph: 02 6620 3569
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Appendix 3 Description of the VIBA

Source: Extracted from Ellemor-Collins and Wright, 2008, p. 107

What Is VIBA?

Videotaped interview-based assessment consists of a dynamic one-to-one interview of a student. The teacher poses mathematical tasks, observes the student's responses, and selects follow-up tasks based on the observations. The teacher may also ask students to explain their strategies. A wide range of mathematical tasks are suitable for an interview-based assessment. Educators who use interview assessments have recommended their potential for decades (Cross and Hynes 1994; Labinowicz 1987; Weaver 1955). Long and Ben-Hur (1991) recommend interview assessments to enable teachers to "see problems through the eyes of the students, to respond to each student's particular needs, and to focus on stages of learning rather than answers."

The VIBA approach is interview-based assessment with two main refinements. First, the interview is recorded on videotape. A camera is positioned on a tripod to capture the student, the teacher, and the desk (see Figure. 1). The assessment of the student's learning is then based on analysis of the recorded interview. Video analysis is particularly effective in collaboration with colleagues and a knowledgeable instructional leader. Second, VIBA species that the basic goal of the assessment is to determine the edge, or limit, of the student's knowledge and strategies. Determining this edge is achieved through attentive observation and questioning during the interview and skilled video analysis afterward.

Figure 1 Pointers for videotaping interviews

- Ensure that you have parental or guardian permission to record the student.
- An additional person to operate the camera is unnecessary.
- Seat the student beside the teacher on the same side of the table.
- Position the camera on a tripod facing them, close (~2m) and high.
- Adjust the frame to capture the student's face and hands, the teacher beside the student, and the desk in front.
- Take care with lighting: Light the faces and avoid shining light into the camera.
- Take care with sound quality: Ensure that the student's voice is audible and minimize noise from air-conditioning, other people, etc.
- Label videotapes immediately with interview date, student's name, and school name.

Appendix 4 P-4 Learning Framework in Number (LFIN) Models

P-4 Learning Framework in Number (LFIN) Models

P-4 Mathematics Intervention Specialist Project

Early Number

Stages of Early Arithmetical Learning (SEAL)

0. Emergent Counting
1. Perceptual Counting
2. Figurative Counting
3. Initial Number Sequence
4. Intermediate Number Sequence
5. Facile Number Sequence

MN / Early Number

Number Word Sequences (NWS)

Forward (FNWS)

0. Emergent FNWS
1. Initial FNWS up to ‘ten’.
2. Intermediate FNWS up to ‘ten’.
3. Facile FNWS up to ‘ten’.
4. Facile FNWS up to ‘thirty’.
5. Facile FNWS up to ‘one hundred’.
6. Facile FNWS up to ‘one thousand’.

Backward (BNWS)

0. Emergent BNWS
1. Initial BNWS up to ‘ten’.
2. Intermediate BNWS up to ‘ten’.
3. Facile BNWS up to ‘ten’.
4. Facile BNWS up to ‘thirty’.
5. Facile BNWS up to ‘one hundred’.
6. Facile BNWS up to ‘one thousand’.

MN / Early Number

Numeral Identification (NID)

0. Emergent numeral identification
1. Facile with numerals to ‘10’
2. Facile with numerals to ‘20’
3. Facile with numerals to ‘100’
4. Facile with numerals to ‘1 000’
5. Facile with numerals to ’10 000’

Middle Number / EN

Structuring Numbers 1 to 20 (SN20)

0. Emergent spatial patterns and finger patterns.
1. Initial spatial patterns and finger patterns.
2. Small doubles and partitions of 5 (no setting).
3. 5-plus and partitions of 10 (no setting).
4. Formal addition (whole ≤ 10).
5. Formal addition (parts ≤ 10).
6. Formal addition & subtraction (whole ≤ 20).

Note: For levels 1-6, students must use facile strategies, that is, not counting by ones.

Middle Number

Conceptual Place Value (CPV)

0. Emergent inc/decrementing by ten
1. Inc/decrementing by 10 off the decuple with materials
2. Inc/decrementing by 10 formal to 100
3. Inc/decrementing by 10 formal to 1 000

Middle Number

Addition and Subtraction to 100 (A&S)

0. Emergent addition & subtraction to 100
1. Add up from/subtract down to decuple
2. Add up to/subtract down from decuple –small
3. Add up to/subtract down from decuple –large
4. Add/subtract across a decuple
5. 2-digit addition with regrouping
6. 2-digit addition and subtraction with regrouping

Note: For levels 1-6, students must use facile strategies, that is, not counting by ones.

Middle Number

Multiplication and Division (M&D)

0. Emergent grouping
1. Initial grouping
2. Perceptual counting in multiples
3. Figurative composite grouping
4. Repeated abstract composite grouping
5. Multiplication and division as operations

Appendix 5 Independent Auditor Report

Southern Cross
UNIVERSITY
A new way to think

Letter of Attestation
School of Education, Southern Cross University

Independent Auditor's Report
A thesis submitted for the award of the degree of Doctor of Philosophy on the title:
Targeted one-to-one instruction in whole-number arithmetic: A framework of Key Elements.

This letter is to verify that I have acted as independent auditor for Thi Le Tran on her doctoral research project on mathematics intervention teaching. I am a researcher in the School of Education at Southern Cross University. I have had extensive experience with the instructional approaches in Ms. Tran's project, and with the Mathematics Intervention Specialist Project from which the data is drawn. I also have research expertise in the close analysis of instruction from video recordings, such as carried out in her research. Ms. Tran asked me to verify the authenticity of her data collection and analysis processes, which I agreed to do.

I am familiar with the ethics committee approval and requirements for the research. The data collection and analysis processes are consistent with the approved research proposal.

The body of video recorded lessons used is a rich and fascinating data set. The quantity and quality of data are certainly adequate. The videos record authentic one-to-one intervention instruction, as I have witnessed and experienced in long-term intervention teaching. I have checked a selection of the recorded lessons, and the transcriptions are accurate.

I have checked the coding of a selection of the transcriptions on NVivo. Data was coded according to key elements of the instruction. I find the descriptions of the key elements have a convincing fit with the samples of data, and the coding of the key elements make reasonable and consistent interpretations of the data. An example scenario of instruction has been chosen for each key element, and I find these examples authentic and representative of other selections of data.

In summary, I can verify that the video data is authentic and substantial, that the data was transcribed and analysed according to the procedures outlined in the methodology chapter of the thesis, and that the findings are reasonable, accurate, and representative of the instruction in the data.

29/1/2016

David Ellemor-Collins
School of Education
Southern Cross University
Lismore, Australia

Paul Wardrop JP No: 104450
Justice of the Peace in and for
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Appendix 6 A Section of the Reflective Journal

Following are:

(i) a piece of work recorded when identifying the 'Teacher problematic behaviours'; and

(ii) a discussion of the link between 'positive infidelity' (Munter, 2010) and the Key Elements.

* Prohibited teacher behaviors: refers to behaviors ~~which~~ that ~~the~~ teachers are ordered not to use during teaching.

(a) directing demonstrating a method for solving a problem (rather than allowing the student to develop~~ment~~ and use his/her own strategies).

including:

① Interrupting the student.

② Rushing or indecent haste

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(b) eliciting behaviors from the student (e.g., entering into guessing games in which the student attempts to be effective by figuring out what the tutor has in mind rather than by thinking through a problem)

③ (c) Giving unwarranted response refers to a teacher's response which is not called-for under the given situations.

including: ① Red-herring

② inappropriately reposing

③ providing unnecessary support

④ Miscuing.

Ask: Anser_focus → in Munter
Strategy - focus
① Level 1, 2, 3 addition cards

(d) others:
① Giving back-handed compliment
② Making multi_request at the same time
③ Flagging a Task as being difficult.

* Positive infidelity (posing) was to include potentially effective adaptations that are inconsistent with the MR model.
① Re-voicing: teacher re-voices student's strategy
→ ≈ Recapulating
② Different strategies: Teacher asks student to solve problem in a different way.
→ Querying to a correct response (Sometimes) Explaining (Sometimes)
③ Compare strategies: Teacher ~~states~~ questions encourage student to examine the mathematical similarities and differences among two or more strategies.
④ Some KE in the new list
① Recapulating
② Confirming and highlighting a correct res
③ Stating a goal
④ Asking "what do you notice"

Appendix 7 Assessment Analysis Sheets

Pre- and post-Assessment Data 2011

Teacher	Student	Year Level	One-Minute Tests (%)				SINE			Interview 2 or 3B	Interview 3A			Interview 3B	Interview 3C	Interview 3D
			+	–	×	÷	%	month	Test	SEAL	FNWS	BNWS	NID	SN 1-20	CPV	A&S100
Amilia	Mia Pre		48	27	3	6										
	Post		73	42	52	24										
Amilia	Kate Pre		24	18						3	3	3	1	1		
	Post		30	21						2	5	5	3	1		
Emma	Hannah Pre	3								3	5	5	3	2	0	0
	Post									4	6	6	4	5	3	3
Sophia	Ben Pre	4	24	21	9	6	29	Feb	B		3	1	4	1	0	1
	Post		67	21	27	15	35	July	B		6	5	5	5	2	1
Sophia	Chloe Pre	3	21	21	0	3	39	Feb	A		1/ 3	1 / 3	4 / 4	1 / or 2	0	No-Pre test
	Post		55	33	27	12	48	July	A		6 / 6	5 / 5	5 / 5	4	2	2
Ava	Ella	3	27	45	0	0	36	Feb	3/4A		3	3	3	1	0	
	Post		60	48	15	55	52	June	3/4A		6	5	5	4	2	1

Appendix 8 Learning Domains

The term of ‘domains of arithmetic knowledge’ (B. Clarke, McDonugh, & Sullivan, 2002; Dowker, 2004; Wright et al., 2007) is used in this investigation to indicate some substantial coherent topics in whole-number arithmetic. This investigation focuses on four domains including: A-Number words and numerals; B-Structuring numbers 1 to 20; C-Conceptual place value; and D-Addition and subtraction to 100. Brief description of each domain is presented as follow. The information are extracted from Wright, Stanger et al. (2006).

Number words and numerals involves knowledge of basic number word sequences and numerals sequences in the range to 1000 and beyond, including sequences by 1s, 10s, 100s, and by other multiples such as 2s, 3s, 4s and 5s. This domain also involves reading and writing numerals, up to 5-digit numerals and further.

Structuring numbers 1 to 20 involves number combinations and partitions in the range 1 to 20, and facility with mental strategies for addition and subtraction the do not involve counting by ones. This domain includes the significant sub-domain of structuring numbers 1 to 10.

Conceptual place value involves flexibly incrementing and decrementing numbers by 1s, 10s and 100s. This formal knowledge of ones, tens and hundreds is foundational in mental strategies for multi-digit computation, and can be distinguished from conventional place value knowledge.

Addition and subtraction to 100 involves facility with mental computation for addition and subtraction in the range to 100, and beyond. The domain includes the sub-domain of higher decade addition and subtraction.