

Achieving new heights in Cumbria: Raising standards in early numeracy through mathematics recovery

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Abstract

This article describes how standards in early numeracy were raised within Cumbria by the application of the Mathematics Recovery Programme. It reports data showing how children's numeracy improved as a result of the programme, and describes effective elements of the in-service teacher training programme which was implemented. This work is an example of how teachers and educational psychologists can work together to develop and disseminate good practice in teaching, which is based on a sound theoretical and evidence base. Mathematics Recovery (MR) is an evolving, research-based programme which was first developed in the 1990s, in order to meet the needs of children who were not reaching age-related expectations for numeracy skills. There is an underlying model of how children acquire strategies and numerical knowledge, and an explicit set of principles of good teaching. The MR materials include short-term, intensive, individual teaching programmes, as well as group and class teaching. The paper reflects on the nature of 'best practice in assessment for learning' (DFES 2005b), which we argue is dynamic in character, in that the assessment is embedded in the teaching, with the assessor playing a mediating role, supporting the learner to construct and elaborate their own model of number. We show how Mathematics Recovery implements this approach to assessment and teaching, through the design, implementation and evaluation of programmes of short-term intervention. The paper concludes with some evidence of how the above has impacted on teachers' professional development and changed classroom practice in Cumbria.

Support for numeracy difficulties

Within the British educational system, there has traditionally been more emphasis placed upon addressing children's literacy difficulties, than on addressing their numeracy difficulties. This is reflected in the resources which Local Authorities have allocated to the two areas: in many Authorities there will be a team of specialist literacy teachers to address Special Needs in literacy, but no corresponding team for such needs in numeracy. However, we do have evidence both of the existence of children's difficulties with numeracy, and of the disadvantage which people with poor numeracy skills suffer in adult life. Since the introduction of the National Numeracy Strategy's 'Framework for teaching mathematics' in 1999, the number of children achieving the target level for their age in mathematics at the end of Key Stage 2 has increased by 16 per cent

(DfES, 2004 and 2005a). But there is still a long tail of children who do not achieve the target level, with a steady proportion of children achieving below level 3, between 2001 and 2004 (DfES, 2005b). The long-term consequences of numeracy difficulties are serious: research suggests that, amongst adults, poor numeracy is more disadvantageous in the labour market than is poor literacy (Basic Skills Agency, 1997).

There is, then, a pressing need to address this area. The Primary National Strategy has responded through its model of 'waves' of intervention: Wave 1 being high quality learning and teaching for all in daily lessons; Wave 2 being targeted, short term small group interventions; and Wave 3 being a more individualised, short term intervention to address 'fundamental errors and misconceptions' (DfES, 2005b). Wave 3 is intended for pupils in Key Stage 2. Thus, whilst Wave 2

might be regarded as being early intervention to prevent children from falling behind, Wave 3 is targeted at children who have made unsatisfactory progress across the three years of infant-aged schooling.

In 2002 the Cumbria LEA piloted and then implemented the Mathematics Recovery programme which complements all three waves of intervention. In its original form, it was a short term, intensive individual programme for pupils in Year 1 of Key Stage 1, and thus constituted intervention to prevent failure, earlier than Wave 3 and more intensive than Wave 2. There are not many such documented intervention programmes available for the Key Stage 1 age group. In her recent review 'What Works for Children with Mathematical Difficulties', Ann Dowker reviews MR very positively, and cites it as one of the two available large-scale, individualised, componential programmes based on cognitive theories of arithmetic (Dowker, 2004). As will be seen below, the MR programme has also been developed so that it can be applied more widely than just as an individual programme, and it can be used as part of Wave 1, Wave 2 and Wave 3.

The key features and origins of the mathematics recovery programme

The key features of MR can be summed up under four headings – Early Intervention, Assessment, Teaching and Professional Development. The assessment and teaching strands use a strong underpinning theory of young children's mathematical learning which leads to a comprehensive and integrated framework for both assessment and teaching. The programme has a detailed approach to, and specific diagnostic tools for, the assessment of children's early number strategies and knowledge. Following the assessment, teachers can employ an especially developed instructional approach and distinctive instructional activities which can be applied to individuals in small-group or class situations. The programme also has an intensive, short-term teaching intervention for low-attaining 5–8-year-old children by specialist teachers. The

entire programme provides an extensive professional development course to prepare the specialist teachers, and ongoing collegial and leader support for these teachers.

MR was originally developed in New South Wales, between 1992 and 1995. It emerged from detailed research studies of how children's number knowledge develops (Wright, 1991; Aubrey, 1993; Young-Loveridge, 1989, 1991). From this, a model of the usual course of this learning was constructed, and assessment tools and techniques were developed, to enable individual children's knowledge to be described in the terms of the model. Wright and his colleagues went on to design an individual teaching approach and materials, intended to move children on through the model, by working in a very detailed way within the child's Zone of Proximal Development (ZPD), that is, planning instruction which is focussed just beyond the child's current levels of knowledge. These individual teaching programmes were evaluated, and shown to be very successful in moving children on through the stages and levels of the model. (Wright *et al.*, 1994; Wright *et al.*, 1998).

The approach has been further developed into its current, published form (Wright, Martland & Stafford, 2006; Wright, Stanger, Stafford & Martland, 2006; Wright, Martland, Stafford & Stanger, 2002). Materials now include assessment tools, teaching programmes for individual children and a book on using the approach in classroom teaching. MR is now in wide, international use, in Australia, the USA, New Zealand, Canada, the UK and Ireland.

Assessment in the mathematics recovery programme

Mathematics Recovery involves a distinctive approach to assessing young children's numerical knowledge. The origins of this method are in research projects conducted in the 1980s and 1990s that focused on understanding children's numerical strategies for addition and subtraction and the modifications children make to their strategies

over time (e.g. Cobb & Steffe, 1982; Steffe & Cobb, 1988; Steffe *et al.*, 1983; Wright, 1989, 1991a). In 1998 the approach was extended to include a focus on children’s early multiplication and division knowledge. This work drew on an extensive range of research (Steffe, 1992b; Steffe & Cobb, 1988; Steffe, 1994; Mulligan, 1998) and the Count Me In Too project (NSW Department of Education and Training, 1998).

The assessment in MR is distinctive on two counts. First, it is interview based and second, the assessment interview is videotaped so that the teacher does not need to record the child’s responses during the course of the interview. The benefit of not having to make notes is that the assessor is free to observe, listen and engage in questions with the child in order to detect the most sophisticated strategy the child uses.

Underlying the development of MR is a belief that in early number learning it is very important to understand, observe and take account of children’s knowledge and strategies when solving tasks. Children’s early numerical knowledge varies greatly and their strategies are multifarious. Thus, across children, early numerical knowledge is characterized by both commonalities and diversity. As indicated by the research of Denvir & Brown (1986a, 1986b), it is insufficient to think that every child’s early numerical knowledge develops along a common developmental path. For example, one important factor in a particular child’s developmental path, it is believed, relates to the nature of the settings in which the child’s prior learning has occurred. Also, children who may appear to an observer to be in the same setting, or learning situation, will construct the situation idiosyncratically and thus different kinds of learning are likely to occur.

The child’s process of constructing numerical knowledge can be thought of in terms of progression or advancement. Children reconstruct or modify their current strategies and doing so is nothing more or less than progression, advancement or learning. Given this, it is useful to consider the notion

of the relative sophistication of children’s strategies. For example, the child who has no means of working out nine plus three other than counting out nine counters from one, counting out three counters from one, and then counting all of the counters from 1 to 12, is using a far less sophisticated strategy than the child who ignores the counters and says nine plus three is the same as ten plus two, and I know that is 12 without counting. Understanding the progression of the strategies which children use in early number situations is the key to advancing teaching staff’s professional knowledge and learning. We refer to the progression as SEAL (Stages of Early Arithmetical Learning). They are:

Stage	Significant tasks
Stage 0:	Emergent Counting
Stage 1:	Perceptual Counting
Stage 2:	Figurative Counting
Stage 3:	Initial Number Sequence
Stage 4:	Intermediate Number Sequence
Stage 5:	Facile Number Sequence

Assessment tasks as a source of instructional activities

Virtually all of the assessment tasks are ideally suited for adaptation to instructional activities. Further, because the assessment tasks are organised into task groups, the tasks within a task group or across several groups typically constitute an implied, instructional sequence. Again, although the tasks are presented in a format for one-to-one interaction, they are easily adapted to situations involving small or large group instruction.

Implementation of mathematics recovery in Cumbria

Cumbria began its involvement with MR on a small scale in 2002 with a group of eight teachers, two Numeracy Consultants and one Educational Psychologist. Subsequently, a MR Team consisting of a Numeracy Consultant (two days per week), an Educational

Psychologist (half a day per week) and three teachers (one day per week each) has worked to support and develop the use of MR within the County.

A major focus is the running of an annual course to train teachers and teaching assistants. The course takes place over two terms, with assessment being covered in the first term, and the teaching programme in the second term. There is a total of seven centre-based training days, with two or more tutor visits to participants' schools. During the course, participants engage in video-taped practice assessments, and design and run a teaching programme with an individual child. So far, 97 schools have undertaken the training, which represents almost one third of the primary schools in Cumbria. (The training has also been found useful by some special school and secondary school teaching staff). Schools have been encouraged to send a teacher and teaching assistant on the course together, in order to promote the use of the programme later in school at both the classroom and individual child level. The teachers and assistants have worked closely together on the programmes for children, and have found this particularly helpful in the development of their skills.

Staff who successfully complete the MR training are able to apply for funding to run individual MR programmes, for pupils whom they have assessed as functioning well below the expected levels on the MR assessments. The effectiveness of these programmes is evaluated, through analysis of pupils' results

on the MR assessments before and after the programme. Most pupils make gains of two SEAL stages (e.g. they move from having to see and count concrete objects in order to add two sets (Stage 1 on SEAL) to being able to work without visible objects and to 'count-up-from' and 'count-down-from' to solve addition and subtraction problems, including missing addends and missing subtrahends (Stage 3 on SEAL)). They also increase their ability in other aspects of number: saying forward and backward number word sequences, to identifying numerals and recognising spatial patterns. Indeed, so far the small number of pupils who have not made a gain of at least one SEAL stage during their MR individual programme have all made measurable gains in these other aspects. See Table 1 below, for a summary of the gains in SEAL stages made by 2 1 0 Cumbrian pupils who received individual programmes between April 04 and March 06.

These increases are similar to those reported in the Australian MR research, although the Cumbrian programmes are shorter in duration (about 20 sessions, half an hour each, taking place three or four times a week – whereas the Australian programmes were more than twice this length). As Ann Dowker says, 'Relatively small amounts of individual intervention may make it possible for a child to benefit far more fully from whole class teaching' (Dowker, 2004). Evaluating how well the children have generalised and adapted the learning is more difficult.

	No stages gained	1 stage gained	2 stages gained	3 stages gained	4 stages gained
April 04 – March 05 100 pupils	9	26	44	19	2
April 05 – March 06 110 pupils	4	31	57	12	6
% of total	6.2	27.1	48.1	14.8	0.04

Table 1: Gains in SEAL stages made by 210 Cumbrian pupils who received individual programmes (Holliday, 2005; Holliday, 2006)

Informal teacher reports, collected on tutorial and other visits to schools, consistently say that the pupils are performing better in class, and that they have gained in confidence and independence. It will soon be possible to analyse the mathematics SATs results from the end of Key Stage 1, tracking those pupils who received a funded programme, and comparing results with comparable pupils who did not.

In addition to staff training and monitoring of individual programmes, the MR team are developing other ways of supporting the use of MR in schools. These include the publication of guidance for schools on the use of MR for group work and for work in the Foundation Stage; work to develop ICT materials for whole class use; work with teaching staff on using MR in the daily mathematics lesson; establishing regular support meetings to update staff and for them to share developments.

The development of the MR work within Cumbria would not have been possible without dedicated funding for the project. This was not initially available. However, following the evaluation of the success of the first cohort of training, it was possible to argue the case for some of the existing Special Needs resources to be directed towards numeracy. Currently, there is an annual budget which pays for the salaries of the MR Team, and for the delivery of some individual programmes to pupils in schools.

Why does MR work so well?

Our ongoing evaluation of the work suggests that MR is highly effective, including with children who have already received Wave 2 interventions, in the form of small group work based on National Numeracy Strategy Springboards, or on National Numeracy Strategy objectives which had been tracked back to earlier curriculum stages. Yet many of the MR teaching activities resemble those in other programmes, and cover ground which is also in the Numeracy Strategy. So why is it so successful, especially when used

with pupils who have some history of mathematics difficulty?

A large part of the answer to this, we believe, lies in the way in which the assessment and teaching are used together, within a framework which is constructivist in its nature. The assessment is not seen just as a measure of what has been learned, but as an integral and ongoing facet of the teaching, which will inform both what is taught next, and the approach and materials which are used in the teaching. Although the initial assessment does give summative information about the levels and stage at which the child is functioning, its central purpose is to allow a qualitative, detailed analysis of the strategies the child is using. The assessor presents the child with a problem, observes the child working, and explores the child's responses (through questioning and judicious presentation of new problems), to find out how the child thought whilst solving the problem. This information is recorded (after the assessment interview) in a Pupil Profile, which highlights the child's present strategies, strengths and weaknesses, and possible next steps for development. This profile is then used to design the teaching programme for the pupil, drawing on the range of available teaching activities within the MR materials.

This approach to assessment continues throughout the teaching programme, as the child's responses during teaching are observed closely, and used to guide the next teaching steps. The aim is always to be working within the child's Zone of Proximal Development (Lunt, 1993; Lidz, 1995), so that the child succeeds, with small but well-targeted prompts from the MR teacher. The MR teacher role is critical here: it is to select appropriate problems for the child, present them in a suitable setting, support the child successfully to find their own solution to the problem and help the child to reflect on what they are doing. This is not didactic teaching: modelling of solutions rarely happens, and when it does is usually associated with the learning of basic facts (such as the words in the forward number word

sequence). This is a mediating role, and is similar to the teacher role in the Dutch 'Realistic Mathematics' work, which refers to pupils as engaging in 'guided reinvention', and stresses the importance of knowledge being constructed by the child. (Gravemeijer, 1994; Milo, Ruijssnaars & Seegers, 2005).

The problem-centred approach is used, within MR, as a very important tool for ensuring that the teaching remains constructivist in its orientation. A central aim of the MR programme is for children to develop their own, increasingly powerful concepts of number, which they will be able to use as a basis for subsequent learning (Cobb & Merkel, 1989). Because they have been developed by the children elaborating their concepts in the course of their own problem-solving, these constructs will be fully understood by the children, in a relational, rather than only an instrumental, manner (Skemp, 1976). Thus, the children will not merely be following a learned 'recipe' for solving a particular, familiar type of problem (showing instrumental understanding), but will be able to devise their own strategies and algorithms for solving novel types of problem (showing relational understanding). Such a 'shift from procedures to reasoning' (Wheatly & Reynolds, 1999) is essential, if children are to become confident and independent learners who will be able to generalise and extend their knowledge in new contexts. The problem-centred approach fosters this relational understanding, through 'developing a setting in which children can invent and discuss their own strategies' (Cobb & Merkel, *op cit*). In an individual MR session, this will be done by presenting the child with a problem which is slightly more difficult than those which the child has previously solved, and allowing as much time as is necessary for the child to work on the problem. The teacher will observe closely, and use their knowledge of what strategies and concepts that child already possesses, to offer prompts that will lead the child towards developing more sophisticated strategies. Importantly, the child will be encouraged to

check their solutions, by using less sophisticated strategies. This will enable them to build links between their developing concepts, so that they are continuously elaborating their mental model of the number system, through solving the problems. As Wright expresses it, 'for the constructivist teacher, advances in the children's knowledge occur when the children modify their current ways of operating in response to a problematic situation.' (Wright, 1990).

During the teaching sessions, the MR teacher continues to observe and assess the child's responses, with a strong focus on how the child makes use of the support which the teacher offers. The teacher is continuously making and testing hypotheses about what experiences will now help the child to develop further their models of number. Thus, throughout the programme, the teacher can be regarded as engaging in Dynamic Assessment (Lidz, 1995; Elliott, 2003). The role of the teacher is to mediate the child's learning experience, by locating the child's Zone of Proximal Development (ZPD) (i.e., the region where the child can only succeed with some support), and working with the child in this Zone. An example from a teaching programme may serve to illustrate the style of the teaching, showing how the teacher mediates the child's learning, choosing different prompts and settings, in order to help the child to construct her responses. The section of dialogue and commentary below comes from an early session in a programme with Gertie, a girl aged 6 years and 5 months.

**Programme extract: Session 2:
work on forward number word
sequence to 20**

Gertie has been doing FNWS successfully, although she hesitated at 12. The teacher presents the Number Word After task, and Gertie succeeds with 5, 15, 24, 29 and 7. The teacher then presents 12, and the following dialogue happens:

T: What number comes straight after 12?

G: 11

T: we're going forwards, so it's the number that comes just after 12.

G: 14

T: 12.....? (re-presents task, and waits)

G:13

T: Good. How did you do that?

G: I just thought in my head it was.

T: Did you count? (G shakes her head.)

G then succeeds with NWA 3 and 12

T: 19

G: 18

T: 19, what comes next, the number just after 19? (re-presents task)

G:17

At this point, G's body language is interesting. She is slumped in her chair, with her hands up around her mouth, beginning to squirm. She seems bored or tired, and uncomfortable. She seems to be signalling that she has had enough of this hard task, but T persists:

T: You're going backwards, you're doing the numbers before. If you were counting forwards, what would come just after 19?

No reply from G, after a long pause. T decides to try to get G to count forwards, and listen to her own voice saying the number after 19:

T: Can we do some counting forwards... we'll start at 18. 18...

G: 19

T: 20

G: 21

T: OK, so what comes just after 19?

G: 21

T: 19...?

G: 19.....

This has not worked. T brings out a numeral track, from 11 to 20. T points to each number, and G reads them out correctly.

T: Where's 19?

G points to 19

T: What comes just after it?

G: 20 (Points to it.)

T puts numeral track away.

T: What comes just after 19?

G: 20

T: Good. What comes just before 20?

G:

T: If you were counting, what would come just before 20?

G:19

T: Well done!

In this extract, Gertie eventually succeeds with NWA for 12 and 19, but with a lot of difficulty, and with external prompting. Future sessions will show that her counting skills vary a lot, from day to day. She is also worse in the afternoons, when she seems tired, and has more difficulty in concentrating. Several interesting points emerge, from this extract. Although Gertie can produce the Forward Number Word Sequence beyond 20, she has not connected this knowledge to the Number Word After task. The teacher attempts to help her make that connection. First, this is done verbally, by encouraging her to count. (She uses a series of graduated prompts: Did you count?... If you were counting forwards, what would come just after 19?...do some counting forwards....) The teacher observes carefully, and decides to fine tune the teaching, through the choice of problems. (Having discovered a difficulty with 12, she works on this, then asks a different question, before revisiting 12 to check. The difficulty with 19 is then worked on, until this is resolved. At this point, Gertie has had enough of this task, and the teacher moves on to a different Key Topic.) This teaching is in Gertie's ZPD: she cannot do it independently, but eventually gets there, with teacher mediation.

Perhaps the most powerful and positively evaluated session in the teaching course is one which is run as a class tutorial, where teaching staff bring scenarios from their ongoing teaching programmes and ask the group to help them with teaching ideas (i.e., ideas for mediating the child's experience). This leads to exchange of ideas and rich discussion, focussed on how to set up an experience which will support that particular child to solve the problem in question. At this point in the course, it becomes evident that not only are the teaching staff working within the children's ZPD, but that they are also developing their teaching skills through working within their own ZPD, using the tutors and each other for support.

Beyond individual programmes

In its original form, MR was delivered to children as short term individual programmes, targeted at Year 1 aged children who were beginning to fall behind their peers in

numeracy. However, it has developed considerably beyond this. For example, in New South Wales and in New Zealand the approach is used across the first three years of schooling, as a framework for the teaching of numeracy to all pupils. This implementation is called Count Me In Too (CMIT), and does not deliver individual programmes to children. It has been evaluated (through pre and post assessments of pupils, as well as questionnaires and case studies with teaching staff and facilitators involved in the programme) as highly successful, both in promoting pupil progress and in increasing teacher knowledge and understanding (Thomas & Ward, 2001). Key elements in the success of this group approach were found to be: increased teacher understanding of how children learn number; increased focus on the strategies which individual children actually use; the availability of assessment tools which can be used to group children appropriately for working on particular learning objectives (Thomas & Ward, *op cit*).

The Cumbrian experience also shows that the effectiveness of MR goes well beyond the individual programmes. Examples of this include:

- Teaching staff who attend the training often respond to the assessment course by spontaneously implementing changes in the way they deliver their class teaching. They come to the second and subsequent training sessions keen to talk about changes they have already made. Many focus initially on the 'mental and oral starter' part of their lessons, noticing that most children are working in their ZPD for only a small part of this activity. They then find different ways to organise and present the activity, so that it is better differentiated to match the children's needs.
- Teaching staff raise their expectations of what children can achieve.
- About half of the teaching staff who complete the course apply for funding to run individual programmes. (In the financial year 2005–2006, funding was approved

for 111 programmes, and 93 per cent of those programmes have so far been completed.) However, many of the remaining teaching staff use the approach to make changes in their classroom teaching, to organise groups of children for teaching, or to deliver teaching to small groups.

- Some teaching staff are using the assessment materials to track children's progress throughout the infant school, and to pinpoint the need for specific interventions with particular children, or for staff training in particular areas of numeracy teaching.
- Teaching staff feel that their expertise in early numeracy is recognised within their school and feel that their skills are used to advantage, as colleagues consult them regarding children's progress and the construction of teaching strategies.
- The teaching staff report that they can readily use existing classroom materials but now in a more effective way: they try to promote mental strategies through providing a set of integrated activities in multiple settings.

Although MR can be very effectively applied in whole class and small group contexts, the MR Team take the view that, in initially learning to apply the principles of MR, it is extremely effective to work with an individual child. This allows the teacher to work continuously at finding where the child is, and developing ways of supporting them to move on. This cannot be done so precisely when working with a group, where there is often the need to make a compromise, or move on before one child is really ready.

The MR Team have formed the impression that many teaching staff develop their knowledge, understanding and practice of numeracy teaching considerably, through using MR. A research project is attempting to explore these changes, through in-depth Personal Construct Psychology interviews with staff (Willey, ongoing). Results so far suggest that staff constructs about teaching of number change markedly, following

training and use of MR. Changes to constructs are in line with the underlying principles of MR, and include the following:

- Trying to take pupils back to first principles so they can build understanding, rather than trying to plug gaps in their procedural knowledge
- Having an understanding of how children develop number knowledge
- A growing commitment to promoting pupils' independent learning
- A belief that good teaching will be enjoyable and motivating for pupils
- Willingness to wait whilst pupils think, and to observe closely what they do
- A belief that children will learn effectively, if they are given tasks within their ZPD and a small amount of support
- A view of the teacher as facilitator and guide, rather than transmitter of knowledge.

The staff who were interviewed were asked to rate themselves on the construct 'teaches numeracy very well', for both before and after the MR training. All but one of them felt that they had improved. Even staff with many years of experience felt that engaging with MR had moved their teaching on significantly.

It seems, then, that teaching staff do develop skills and knowledge through implementing MR, and that they do put this into practice in their subsequent work with pupils. This is because MR provides a structure within which staff feel safe to experiment with a more constructivist approach. The teaching activities given within the programme function as examples. Although it would be possible to run individual programmes using only the teaching activities given, this does not generally happen: the teaching staff adapt, tailor and extend the activities, to address more exactly the needs and interests of each child. The teaching staff are able to do this because they have learned to use the principles of MR, to generate their own solutions to new situa-

tions. The teaching staff have gained confidence in pupils' abilities to learn in a constructivist way, and in their own abilities to guide such learning.

Conclusion

We have shown how one Local authority has implemented Mathematics Recovery and evaluated its impact. Individual pupils who receive MR programmes make good progress in basic numeracy skills. Teachers and teaching assistants develop their knowledge, skills and confidence to teach numeracy. The Maths Recovery principles, assessment tools and activities work well at a number of levels: in individual programmes, in group work and in informing good classroom teaching.

The greatest power of Mathematics Recovery lies in its use as a tool for the professional development of teaching staff. Staff who engage with Mathematics Recovery develop an enhanced faith in pupils' ability to learn and to solve problems for themselves. Alongside this, they become more confident in their own ability to assess where pupils are, and to offer appropriate support to help pupils learn. For many staff, this results in a significant shift in their teaching style, away from the didactic and towards a more pupil-focused, constructivist outlook.

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